INTRODUCTION.

defn :- An equation involving one dependent variable and its derivative with respect to one or more independent variable is called a differential equation.

eg:- 1) $(2x+3y)\frac{dy}{dx}+(7x^2-8y)=0$

2) $y''+(8x+3)y'+e^y \sin x=0$.

3) $y\frac{\partial u}{\partial x}-x\frac{\partial u}{\partial y}-z^2\frac{\partial u}{\partial z}=0$ (P.D.E)

defn :- Order.

The order of a differential equation is the order of the highest derivative occuring in the equation.

degree :- The degree of a differential equation is the degree of the highest derivative which occur in it.

eg:-1 consider $y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y = 0$.

here order - 1 $\left(\cancel{\frac{dy}{dx}}\right)$

degree - 2.

eg 2:- $(y^2+x)\frac{d^2y}{dx^2} + 2y\left(\frac{dy}{dx}\right)^2 = 7$.

is of second order and first degree.

Differential equation as mathematical models

imp:

modeling is the process of writing a differential equation to describe a physical situation.

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eg:- Newton's second law., which states that the mass of the object times its acceleration is equal to the net force on the object.

In mathematical terms

$F = ma$, where m is the mass of the object, a is its acceleration and F is the net force exerted on the object.

here the acceleration a is related to v by $a = \frac{dv}{dt}$ here

$F = ma \Rightarrow F = m\frac{dv}{dt}$.

Problems

Find the order and degree of following D E.

1) $(2x+3y)\frac{dy}{dx} + (7x^2-8y)=0.$

2) $y'' + (8x+3)y' + e^y \sin x = 0.$

3) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0.$

4) $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = x.$

5) $\frac{d^4y}{dt^4} + \frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} + \frac{dy}{dt} + y = 1.$

6) $\frac{dy}{dt} + ty^2 = 0.$

7) $(1+y^2)\frac{d^2y}{dt^2} + t\frac{dy}{dt} + y = e^t.$

First order differential equation.

The general form of first order differential equation is $\frac{dy}{dx} = f(x,y)$.

Separable differential equations.

Many differential equation can be reduce to the form:

$$\frac{dy}{dx} = \frac{M(x)}{N(y)} \quad \text{--- (1)}$$

$$\Rightarrow N(y)\,dy = M(x)\,dx \quad \text{--- (2)}$$

such an equation is called separable equation.

By integrating on both sides of (2) we obtain the solution.

eg:- solve the differential equation.

$$y' = (1+x)(1+y^2).$$

given equation is

$$\frac{dy}{dx} = (1+x)(1+y^2)$$

$$\frac{dy}{1+y^2} = dx\,(1+x).$$

$$\int \frac{dy}{1+y^2} = \int (1+x)\,dx.$$

$$\tan^{-1} y = x + x^2/2 + c.$$

$$y = \tan(x + x^2/2 + c).$$

(2) solve the initial value problem

$y' = -2xy$, $y(0) = 1$ also $y(x_0) = y$

$$\frac{dy}{dx} = -2xy.$$

$$\frac{dy}{y} = -2x\,dx$$

$$\int dy/y = \int -2x\,dx.$$

$\ln|y| = -2x^2/2 + c$

$\ln|y| = -x^2 + c$

ie $\cancel{y = e^{-x^2}}$

Also given initial condition $y(0)=1$.

$(y(x_0)=y_0)$

substitute this values of x & y in this soln we obtain the value of c

$\therefore \ln|1| = 0 + c$

$0 = 0 + c$

$\Rightarrow c = 0$.

$\therefore$ The particular solution satisfies the initial value problem is

$\ln|y| = -x^2 + 0$

$\Rightarrow y = e^{-x^2}$

3 Find the general solution of the following differential equation

$$y' = x^2/y.$$

4 $$y' = \frac{3x^2 - 1}{3 + 2y}.$$

5 $$\frac{dy}{dx} = x^2/(1+y^2).$$

6. Solve the initial value problem.

$$y' = \frac{1-2x}{y}, \quad y(1) = -2.$$

Equations reducible to separable form.

Type 1 homogeneous differential equa

A D E $y' = f(x, y)$ is said to be a homogeneous D E if $f(x, y)$ can be expressed as a function of the ratio y/x only.

eg :- solve $2xy\frac{dy}{dx} - y^2 + x^2 = 0$.

solu :-

$$2xy\frac{dy}{dx} - y^2 + x^2 = 0$$

$$2xy\,dy/dx = y^2 - x^2$$

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy} \quad \text{—— } (1)$$

Substitute $y = vx$ and its derivative in ~~(1)~~ (1) we get

$y = vx$ (v is another func Then pdt rule)

$dy/dx = v + x\,dv/dx$

$$v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2xvx}$$

$$= \frac{\cancel{x^2}(v^2-1)}{2\cancel{x^2}v}$$

$$x\frac{dv}{dx} = \frac{(v^2-1)}{2v} - v$$

$$= \frac{v^2-1-2v^2}{2v} = \frac{-v^2-1}{2v}$$

$$x\frac{dv}{dx} = -\frac{(v^2+1)}{2v}$$

$$\frac{dv}{v^2+1} \times 2v = \frac{-dx}{x} \quad \text{(separable fu}$$

Now integrate.

$$\int \frac{2v\,dv}{v^2+1} = \int -\frac{dx}{x}$$

$$\ln|v^2+1| = -\ln|x| + c$$

$$\ln|v^2+1| + \ln|x| = c$$

$$\ln(v^2+1)x = c \quad [\because \ln a + \ln b = \ln ab]$$

taking exponential on both sides

we get $x(v^2+1) = e^c$

Replace ~~o~~ v $\left(y = vx \quad v = y/x \right)$

we get

$$x\left(y^2/x^2+1\right) = e^c, \text{ } k \text{ constant}$$

$$x\left(\frac{y^2+x^2}{x^2}\right) = k.$$

$$\frac{y^2+x^2}{x} = k$$

$$y^2+x^2 = kx.$$

Type 2

eqn of the form

$$\frac{dy}{dx} = \frac{ax+by+l}{k(ax+by)+m}$$

eg:- solve the d.e

$$(2x-4y+5)\,y' - (x-2y+3) = 0$$

$$\frac{dy}{dx} = \frac{x-2y+3}{2x-4y+5} \quad \text{--- (1)}$$

$$= \frac{x-2y+3}{2(x-2y)+5}$$

put $u = x-2y$

$$\frac{du}{dx} = 1 - 2\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1-\frac{du}{dx}}{2}$$

Miscellaneous Type

eg. Solve $2x^2yy' = \tan(x^2y^2) - 2xy^2$ — (1)

put $z = x^2y^2$

$\cancel{\frac{dz}{dx}}$ z' or $\frac{dz}{dx} = x^2 y' 2y + y^2 \times 2x$

ie $z' = 2x^2yy' + 2xy^2$

$\therefore$ (1) become.

$$2x^2yy' + 2xy^2 = \tan(x^2y^2)$$

$$z' = \tan z.$$

Problems

1) solve $y' = x^2/y$

2) $$\frac{dy}{dx} = \frac{2y-3x}{2x-y}.$$

3) $$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}.$$

4 $$\frac{dy}{dx} = \frac{y-x}{y-x-1}$$

5 $y' = \tan(x+y) - 1$

4) $\frac{dy}{dx} = \frac{y-x}{y-x-1}$ —①

put $u = y-x$.

$\frac{du}{dx} = \frac{dy}{dx} - 1. \Rightarrow \frac{dy}{dx} = 1 + \frac{du}{dx}.$

Thus ① become.

$1 + \frac{du}{dx} = \frac{u}{u-1}$

$\frac{du}{dx} = \frac{u}{u-1} - 1$

$= \frac{u-(u-1)}{u-1}$

$= \frac{1}{u-1}$

$du\,(u-1) = 1\,dx.$

On integrating.

$\frac{u^2}{2} - u = x + c.$

$$\frac{(y-x)^2}{2} - (y-x) = x + c$$

$$(y-x)^2 - 2y + 2x = x + c$$

$$\Rightarrow (y-x)^2 - 2y = x - 2x + c$$

$$\Rightarrow (y-x)^2 - 2y = -x + c$$

(5) $y' = \tan(x+y) - 1$ —— (1)

put $z = x + y$

$$\frac{dz'}{dx} = 1 + \frac{dy}{dx}.$$

(1) become:

$$\frac{dy}{dx} + 1 = \tan(x+y)$$

$$\frac{dz}{dx} = \tan z.$$

$$\int \frac{dz}{\tan z} = \int dx.$$

$$\frac{1}{\tan z} = \frac{\cos z}{\sin z} dz$$

$$\ln|\sin z| = x + c.$$

$u = \sin z$

$du = \cos z \, dx$

$\int \frac{1}{u} du$

taking exponential.

$$\sin z = ce^{x}$$

$$\sin(x+y) = ce^{x},$$