1. Number System

A number system (or numeral system) is a writing system for expressing numerals, and a mathematical notation for representing numbers of a given set, using symbols in a consistent manner. Ideally, a numeral system will:

- Represent a useful set of numbers (e.g. all whole numbers, integers, or real numbers)
- Give every number represented a unique representation (or at least a standard representation)
- Reflect the algebraic and arithmetic structure of the numbers.

Number systems are basically of two types: non-positional number systems and positional number systems

1.1. Non-Positional Number Systems

In early days, human beings counted on fingers. When ten fingers were not adequate, stones, pebbles, or sticks were used to indicate values. This method of counting uses the non-positional number system. In this system, we have symbols such as I for 1, II for 2, III for 3, IIII for 4, etc. Each symbol represents the same value regardless of its position in the number and the symbols are simply added to find out the value of a particular number.

An example of non-positional number system is the Roman Number System. This number system is quite complicated because it uses many unrelated symbols and there is no zero. The numbers 1 to 10 are represented by I, II, III, IV, V, VI, VII, VIII, IX, X. Twenty is XX, fifty is L, one hundred is C, five hundred is D, one thousand is M, etc.

Since it is very difficult to perform arithmetic with non-positional number system, positional number systems were developed as centuries passed.

1.2. Positional Number System

In a positional number system, there are only few symbols called digits, and these symbols represent different values depending on the position they occupy in the number. The value of each digit in such a number is determined by three considerations:

1. The digit itself.
2. The position of the digit in the number.

3. The base or radix of the number system.

Positional number systems have a radix or base. The *radix* or *base* of a positional number system is defined as the total number of different digits available in the number system.

The number system that we use in our day-to-day life is called Decimal Number System. In decimal number system, the radix is 10. A number system with radix 'r' will have r different symbols. A number in a radix 'r' system would be written as:

$$a_n a_{n-1} a_{n-2} \ldots a_0 . a_{-1}\, a_{-2}\, a_{-3} \ldots a_{-m}$$

and would be interpreted to mean:

$$a_n r^n + a_{n-1} r^{n-1} + a_{n-2}\, r^{n-2} + \ldots + a_0\, r^0 + a_{-1} r^{-1} + a_{-2}\, r^{-2} + a_{-3}\, r^{-3} + \ldots + a_{-m}\, r^{-m}$$

The symbols a_n, a_{n-1} ,a_{n-2},...a_{-m} used in the above representation should be one of the 'r' symbols allowed in the number system. In the above representation 'a_n' is called the most significant digit (MSD) and 'a_{-m}' is called the least significant digit (LSD).

There are two common characteristics for all positional number systems. In all number systems, the value of the base determines the total number of different symbols or digits available in the number system. The first of these choices is always zero. The second characteristic is that the maximum value of a single digit is always equal to one less than the value of the base..

Some of the number systems that are/were in the use are listed in the following table.

Base	Base Name
1	Unary, Primal, Singulary, Henadic, Primary
2	Dual, Binary, Dyadic, Secondary
3	Tertial, Ternary, Trinary, Triadic, Tertiary
4	Quartal, Quaternary, Tetradic
5	Quintal, Quinary, Pentadic, Quinternary
6	Sextal, Senary, Hexadic, Heximal, Hexary
7	Septimal, Septenary, Hebdomadic, Septuary
8	Octal, Octaval, Octavary, Octonary, Ogdoadic, Octonal
9	Nonary, Novenary, Enneadic, Novary, Noval
10	Decimal, Denary, Decadic
11	Undecimal, Undenary, Hendecadic, Unodecimal
12	Duodecimal, Duodenary, Duodecadic, Dozenal
13	Tridecimal, Tredecimal, Triodecimal
14	Quattuordecimal, Quadrodecimal, Tetradecimal

15	Quindecimal, Quindenary, Pentadecimal
16	Sedecimal, Sedentary, Hexadecimal, Sexadecimal

Some of the number systems commonly used in computer design and by computer professionals are discussed below:

1.3. Decimal Number System

In decimal number system, the base is equal to 10 because there are altogether ten symbols or digits (0, 1, 2, 3, 4, 5, 6, 7, 8, and 9) used in this system. In decimal number system, the successive positions to the left of the decimal point represent 10^0, 10^1, 10^2 etc, and to the right of the decimal point represent 10^{-1}, 10^{-2}, 10^{-3} etc. Thus, the column weights of a decimal number are,

$$10^n \ldots 10^3 10^2 10^1 10^0 . 10^{-1} 10^{-2} 10^{-3} \ldots 10^{-n}$$

For example, the decimal number 2586.143 consists of the digit 6 in the units (10^0) position, 8 in tens (10^1) position, 5 in hundreds (10^2) position, 2 in thousands (10^3) position, 1 in 10^{-1} position, 4 in 10^{-2} position, and 3 in 10^{-3} position and its value can be written as:

$$2\times10^3+5\times10^2+8\times10^1+6\times10^0+1\times10^{-1}+4\times10^{-2}+3\times10^{-3}$$

or $$2000+500+80+6+0.1+0.04+0.003 = 2586.143$$

It may be observed that the same digit signifies different values depending on the position it occupies in the number. For example,

in $(2586)_{10}$ the digit 6 signifies $6\times10^0 = 6$
in $(2568)_{10}$ the digit 6 signifies $6\times10^1 = 60$
in $(2658)_{10}$ the digit 6 signifies $6\times10^2 = 600$
in $(6258)_{10}$ the digit 6 signifies $6\times10^3 = 6000$

The principles that apply to the decimal number system apply in any other positional number system. It is important only to keep track of the base of the number system in which we are working.

1.4. Binary Number System

The radix of the binary number system is 2. In this system, only two symbols, namely 0 and 1 are used. Note that the largest single digit is 1, one less than the base. Each position in binary number system represents a power of the base 2. Thus, in binary number system, to the left of the binary point, the right most position is the units (2^0) position, the second position from the right is 2's (2^1) position, and proceeding in this way we have 4's (2^2) position, 8's (2^3) position, 16's (2^4) position, and so on. And to the right of the binary point, all bits have weights that are negative powers of 2. Thus, the column weights of a binary number are:

$$\boxed{2^n \ldots 2^3\ 2^2\ 2^1\ 2^0 . 2^{-1}\ 2^{-2}\ 2^{-3} \ldots 2^{-n}}$$

(Binary point)

This indicates that all bits to the left of the binary point have weight that are positive powers of 2 and all the bits to the right of the binary point have weight that are negative powers of 2 as illustrated in the following example. The decimal equivalent of the binary number 10101.011 is

$1\times2^4 + 0\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0 + 0\times2^{-1} + 1\times2^{-2} + 1\times2^{-3}$

or $16 + 0 + 4 + 0 + 1 + 0 + 1/4 + 1/8$

or $16 + 4 + 1 + 0.25 + 0.125 = 21.375$ in decimal.

In order to be specific about which system we are referring to, it is common practice to indicate the base as a subscript. Thus, we write $(10101.011)_2 = (21.375)_{10}$

The binary digit is often referred to by the common abbreviation 'bit'. Thus, a bit in computer terminology means either a 0 or a 1. A combination of 4 bits is referred to as a Nibble and a combination of 8 bits is referred to as a Byte.

In binary number system, the numbers can be counted as 0, 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111, 10000, 10001, 10010, etc. The following table gives the decimal numbers from 0 to 32 and their binary equivalents.

Decimal	Binary	Decimal	Binary	Decimal	Binary
0	0	11	1011	22	10110
1	1	12	1100	23	10111
2	10	13	1101	24	11000
3	11	14	1110	25	11001
4	100	15	1111	26	11010
5	101	16	10000	27	11011
6	110	17	10001	28	11100
7	111	18	10010	29	11101
8	1000	19	10011	30	11110
9	1001	20	10100	31	11111
10	1010	21	10101	32	100000

We can observe that the use of a smaller base may require more position to represent a given value. For example, $(9)_{10} = (1001)_2$. Hence, four positions are required instead of one position to represent the decimal number 9 in the binary form. In spite of this fact almost all computers use binary system. Reasons for using binary number system in computers are:

1. The first and foremost reason is that the electronic and electrical components, by their very nature, operate in a binary mode. Information is handled in the computer by electronic and electrical components such as transistors, wires, etc., all of which can only indicate two states or condition – ON (represented by 1) or OFF (represented by 0). Transistors are either conducting (1) or non-conducting (0); magnetic materials are either magnetized (1) or non-magnetized (0); a pulse or voltage is present (1) or not present (0) in a wire. Thus, all the information within the computer is represented by the presence or absence of these various signals. The binary number system, which has only two digits (0 and 1), is most suitable and is conveniently used to express the two possible states.
2. The second reason is that computer circuits have to handle two binary digits rather than ten decimal digits. The result is that the internal circuit design of computers is simplified to a great extent. This ultimately results in less expensive and more reliable circuits for computers.
3. Finally, the binary number system is used because everything that can be done with a base of 10 can also be done in binary.

Due to the binary characteristic of electric and electronic circuits, a computer can understand information composed of only 0's and 1's. So, all computers perform their internal manipulations on binary digits. But the programmers use data and other information in the form of decimal digits, alphabets and special symbols. This information is converted to binary codes within the computer because digital computers operate on binary digits.

1.5. Octal Number System

In the octal number system, the base is 8. So in this system there are only 8 symbols or digits: 0, 1, 2, 3, 4, 5, 6 and 7. Here also, the largest single digit is 7, one less than the base. Again, each position in an octal number represents a power of the base, 8. To the left of the octal point, positional values from right most digit are 8^0, 8^1, 8^2, 8^3, etc. and to the right of the octal point, positional values from left most digits are 8^{-1}, 8^{-2}, 8^{-3}, etc. Thus the column weights of an octal number are:

$$8^n \ldots 8^3\ 8^2\ 8^1\ 8^0\ 8^{-1}\ 8^{-2}\ 8^{-3} \ldots 8^{-n}$$

(Octal point)

This shows that all bits to the left of the octal point have weight that are positive powers of 8 and all the bits to the right of the octal point have weight that are negative powers of 8 as illustrated in the following example. The decimal equivalent of the octal number 2057.325 is:

$$2\times8^3 + 0\times8^2 + 5\times8^1 + 7\times8^0 + 3\times8^{-1} + 2\times8^{-2} + 5\times8^{-3}$$

or $1024 + 0 + 40 + 7 + 0.375 + 0.03125 + 0.009765$

or $(1071.416015)_{10}$, *i.e.* $(2057.325)_8 = (1071.416015)_{10}$

In octal number system, numbers can be counted as 0, 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 30, 31, 32, 33, 34, 35, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, etc.

1.6. Hexadecimal Number System

In hexadecimal number system, the base is 16. The base of 16 suggests choice of 16 single character digits or symbols. The first 10 digits are the digits of the decimal number system 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The remaining six digits are denoted by A, B, C, D, E, and F representing the decimal values 10, 11, 12, 13, 14 and 15, respectively. Therefore, in hexadecimal number system, the letters A through F are number digits. The number A has the decimal equivalent of 10 and the hexadecimal number F has a decimal equivalent of 15. Here also, the largest single digit is F, one less than the base. Again, each position in a hexadecimal number system represents a power of the base 16. To the left of the hexadecimal point, each digit from the right most digit have the positional values 16^0, 16^1, 16^2, 16^3, etc., and to the right of the hexadecimal point, each digit from the left most digit have the positional value 16^{-1}, 16^{-2}, 16^{-3}, etc. The column weights of a hexadecimal number are:

$$16^n \ldots 16^3\, 16^2\, 16^1\, 16^0 . 16^{-1}\, 16^{-2}\, 16^{-3} \ldots 16^{-n}$$

(Hexadecimal point)

This shows that all bits to the left of the hexadecimal point have weight that are positive powers of 16 and all the bits to the right of the hexadecimal point have weight that are negative powers of 16 as illustrated in the following example. The decimal equivalent of the hexadecimal number 1AF is,

$$1\times16^2 + A\times16^1 + F\times16^0$$

or $1\times16^2 + 10\times16^1 + 15\times16^0$

or $256+160+15 = (431)_{10}$, *i.e.* $(1AF)_{16} = (431)_{10}$

In hexadecimal number system, the numbers are counted as 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1A, 1B, 1C, 1D, 1E, 1F, 20, 21, 22, 23, 24, 25, 26, etc.

Every computers store numbers, letters, and other special characters in the binary form. There are several occasions (for example, to detect errors in the data) when computer professionals have to know the raw data contained in a computer's memory. A common way of looking at the contents of a computer's memory is to printout the memory contents on a printer. This printout is called memory dump. If the memory dump were printed using binary numbers, the computer professionals have to work with many pages of 0s and 1s. Working with these binary numbers would be very difficult and error prone. Because of the large quantity of the printout that would be required in a memory dump of binary digits and the lack of digit variety (0s and 1s only), octal and hexadecimal number systems are used for memory dump as a shortcut notation for binary.