**QUANTUM MECHANICS**

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Newtonian mechanics, Maxwell’s e. m theory, and thermodynamics guided the growth of science and engineering during the years spanning 17th to 19th centuries. The theories explained all the scientific results of those times and it seemed nothing more could be added. The above theories which are successful in the realms of macroscopic world are regarded as classical physics.

But the classical physics could not explain phenomenon resulting due to interaction of light with matter, such as black body spectrum, photoelectric effect, Compton effect etc. The failure of classical physics (i,e. Newton’s laws, thermodynamics, and wave theory of light) to explain the phenomenon resulting due to interaction of light with matter gave birth to quantum theory of light. Quantum theory was first introduced by Max Planck in the year 1900 while explaining the black body spectrum.

Quantum (wave) mechanics is the department of theoretical physics dealing with the laws of motion of particles in the microcosm region (10-8 to 10-15 m). The objects studied by wave mechanics are crystals, molecules, atoms, atomic nuclei, and elementary particles. In the coming chapters we will discuss the development of quantum mechanics.

**CHAPTER- I**

**PARTICLE PROPERTIES OF WAVES**

By: K.C.A

**Introduction:**

The phenomenon like interference, diffraction and polarization etc. prove that e. m radiation has wave nature. But experiments like Frank – Hertz experiment show that light or e. m radiation is absorbed or radiated in the form of particles known as photons or ‘quanta’ proving thereby that light is of corpuscular nature. **Hence, radiation behaves sometimes as wave and sometimes as particles. This characteristic property is called as ‘dual nature of light’ or particle property of waves.**

In 1900 Max Plank proposed the quantum theory of light to explain the experimentally observed distribution of energy in the spectrum of thermal radiation of a black body. According to Plank’s quantum hypothesis, an atomic oscillator absorbs or gives off energy not in continuous steps, but in integral multiple of certain energy unit **hν** called the quantum. Thus according to quantum postulate the exchange of energy between light and matter is not continuous, but it is in small bundles or packets or quanta of definite energy proportional to the frequency of light. These small packets of energy are called photons.

Thus E α **ν**

or E = **hν**

**E** = **hν**

Energy of a photon

Actually the name photon to the quanta of energy was given by Einstein.

**Black body:**

A body which completely absorbs radiations of all wave lengths incident on it is called a perfectly black body.

Eg: (1) lamp black or platinum black is nearest approach to a perfectly black body.

(2) A cavity in a body connected to the outside by a small hole is a practical black body.

**Blackbody radiation:**



The radiations emitted by a heated black body are called black body radiation or full radiation or temperature radiation.

**Distribution of energy in the spectrum of a black body:**

Lummer and pringsheim studied the distribution of energy in the radiation emitted by a black body at different temperatures.

The radiation from a black body at a particular temp is dispersed by a prism of quartz. The energy of the radiation Eλ for a particular wavelength λ is measured. Eλ is plotted against λ for one particular temperature, say, 904 K. Likewise a number of curves for different temperatures are drawn.

These curves show that the energy is not uniformly distributed and the intensity of radiation is maximum at a particular wavelength.



Eλ

**Rayleigh – Jeans law of spectral distribution of energy :**

According to Rayleigh – Jeans law, the energy distribution in the black body spectrum is given by

Eλd λ = dλ K = Boltzmann constant

Eλ is the energy density in the wavelength range λ and λ +dλ.

This law is found to agree with experimental results at longer wavelengths only but not for shorter wavelengths.

**Ultraviolet Catastrophe**:

We know, Rayleigh – Jeans law for black body spectrum is,

Eλd λ = dλ

According to this law, as we move toward the shorter wavelength side (i.e., toward ultraviolet) the energy density Eλ will increase and finally when,

λ 0, Eλ  ………..(1)

The total energy density for all wavelengths from o to is

E = Eλd λ =

8 = = ………(2)

This result i.e eqn(1) shows that almost all the energy will be confined in vibrations of very small wavelengths. This is known as Ultraviolet catastrophe and is contrary to experimental results. ie, the discrepancy between the Rayleigh – Jeans theoretical conclusions and the experimental result is called Ultraviolet catastrophe.

**Planck’s Radiation Formula** :

Planck was able to deduce a theoretical expression for the energy distribution in black body spectrum on the basis of quantum theory.

According to quantum theory, energy transactions take place in the form of quanta called photons. Each photon has energy **h**  where **h** is Planck’s constant and is the frequency of the emitted radiation. Based on quantum theory and applying the laws of probability, Planck derived an expression for the energy distribution in the black body spectrum belonging to the wavelength range λ and λ+dλ. It is given by

Eλ dλ =

This is Planck’s radiation formula. Planck’s formula agrees with experimental results. It can be shown that the Planck’s law reduces to Wien’s law in the region of shorter wavelengths. Similarly, in the realm of longer wavelengths it becomes Rayleigh – Jeans law.

**Note :**  Black body spectrum cannot be explained on the basis of classical theory. Only quantum theory can explain it. These and other inadequacies of classical mechanics led to the development of quantum mechanics.

**Photoelectric effect :**

Photo electric is the phenomenon of emission of electrons from the surface of metals when light of suitable wavelength falls on them.

or

When e.m radiations such as x-rays, γ-rays, ultraviolet rays or even visible light fall on a metal surface, electrons are emitted. This phenomenon is called photo electric effect. or, photoelectric emission. The emitted electrons are called photoelectrons.

**Demonstration of photoelectric effect** :



A and C are two zinc plates inside a quartz vessel. A is connected to the positive and C is connected to the ‘-ve’ of a battery. If ultraviolet light is allowed to fall on the ‘–‘ ve plate C, there is at once a deflection in the galvanometer, showing that a current is flowing through the circuit. Clearly it shows that electrons are emitted by the plate C and these electros travel up to the plate A. This produces a current through the circuit. This current is called photoelectric current.

**Laws of photoelectric emission**

1. For a given metal and for a given frequency of light, the number of photoelectrons emitted is directly proportional to the intensity of incident light.

2.For every metal, there is a particular minimum frequency of the incident light below which there is no photoelectric emission, whatever be the intensity of incident light. This minimum frequency is called threshold frequency.

3. The velocity and hence K.E of the emitted photo electrons is independent of the intensity of incident light, but depends on the frequency of the incident radiation.

4. Photo electric emission is an instantaneous process. There is no time lag between the incidence of radiation and the emission of electrons.

**Quantum theory of light** : According to quantum theory, the energy carried by a light beam of frequency is concentrated in indivisible packets called ‘photons’

Each photon of light of frequency has the energy h i.e,

E= h

**Photoelectric work function φo:**

For every metal there is a minimum amount of energy to be supplied to liberate the electron from the metal surface. This minimum amount of energy which is just sufficient to liberate an electron from the metal surface is known as the photoelectric work function.

**Einstein’s photo electric equation**: Einstein explained the photoelectric effect by using quantum theory of radiation.When a photon collides with an electron in the metal, it may transfer its entire energy, E = h to the electron. A part of the energy acquired by the electron is used to liberate it from the metal surface and the other part remains with it as its kinetic energy.

The minimum energy needed to liberate the electron from the metal surface is photoelectric work function φo. If it usesminimum energy, ie**,** φo, for ejection, then it comes out with maximum kinetic energy (K.E)

∴ By the law of conservation of energy,

h = φo +(K.E)max

or, (K.E)max = h - φo ……………(1)

At a particular frequency , called threshold frequency, K.E = o

h - φo = o

or φo = h ------------------(2)

∴ (K.E) max  = h - h= h( - )

½mv2max. = h - h

--------------- (3)

This is known as Einstein’s photo-electric equation.

**Nature of light:** The earliest theory for explaining the nature of light radiation was the corpuscular theory of Newton. However, Huygens wave theory could explain satisfactorily the phenomena of interference, diffraction, polarization etc. Thus, these phenomena prove that light possess wave nature. On the other hand, the phenomena of photoelectric effect and Compton effect can only be explained on the basis of quantum theory of light. According to Max Plank’s quantum theory, a beam of light of frequency is composed of tiny discrete packets, each having an energy h. These packets are called photons or ‘quanta’ which behave like particles. Thus photoelectric effect and Compton effect indicate that light possess particle nature. Thus we can say that light possess dual nature, behaving as a wave as well as a particle (photon) ie, light is both wave and particle.

**Wave particle duality** : We know that light is both wave and particle. If there exists something in nature (like light) which has neither purely particle properties nor purely wave properties but has properties of both eg: mass, momentum, wavelength, amplitude, frequency and is neither localized at a point nor extends to infinity, we should call it neither a particle nor a wave. For want of a better name, we simply call it wave –particle. The fact is that there are no particles and no waves in this universe but only wave-particle dualities. To sum up, the concept of wave – particle duality applies to all objects.

**COMPTON EFFECT** : When a monochromatic beam of X-rays is scattered by a substance, the scattered x-rays contain radiation not only of the same wavelength as that of the primary rays but also the radiation of longer wave length.

The unchanged radiation in the scattered beam is known as unmodified radiation or coherent or classical scattering, while the changed radiations are called modified radiation or incoherent scattering. This phenomenon giving rise to modified radiation is called the Compton effect and has been explained satisfactorily on the basis of quantum theory.

**Theory of Compton effect :**  According to quantum theory, the primary x-ray beam is made of photons of energy E= h. These photons travel with the speed of light c and possess, momentum P = E = mc2 = m c .c = Pc. or, P = E/c= h /c



Compton assumed that during the scattering process, the incident photon collides with a free electron in the scattering material.

During the collision, the photon transfers of some of its energy to the electron which recoils with a velocity ‘v’ in a direction making an angle with the direction of the incident photon. The photon itself with reduced energy is scattered in a direction with the original direction.

These scattered photons constitute the scattered modified radiation.

**Before collision:**

1. The energy of the incident photon = h use
2. Rest energy of the electron = moc2 E= mc2 relation,
3. Momentum of the incident photon = h /c assume its velocity c.
4. Momentum of the rest electron o

**After Collision** :

1. The energy of the scattered photon = h scattered

photon

1. Energy of electron = mc2, where m = mass of the electron at

velocity v which is acquired by

it due to recoiling

3)The momentum of the scattered photon =

4)The momentum of the scattered electron = mv, where m =

According to the principle of conservation of energy

+ moc2 =h + mc2 ……………………………(1)

According to the principle of conservation of momentum in the incident direction

+ o = cos + mv cos

According to the principle of conservation of momentum in the perpendicular direction

0 + 0 = sin - mv sin

Form (2), h( - ‘cos) = mv c cos …………………(4)

Form (3), h sin = mvcsin ………………………(5)

(4)2 + (5)2 + =

+ – + =

Or = ……………..(6)

Form (1)

= + h

Square the above eqn.

= +2  + 2h

or = +2h+ ……………..(7)

(7) – (6)

- = (-1)+ 2h+ …………(8)

But L H S of eqn (8) is

- = - m =

=

=

- = ………………..(9)

Put eqn (9) in (8)

= 2()+ 2h+

Or 2h= - 2()

= (

- = (………………….(10)

But, = c/λ and = c, where λ and are the wavelengths of incident and scattered radiations respectively. Put these in eqn (10)

- = (1-)

or = (1-)………………..(11)

This equation is known as Compton shift.

Or, Compton shift  **= = (1-)**………………..(12)

This is the expression for change in wavelength or Compton shift or Compton equation. This equation shows that the increase in wavelength or Compton shift is independent of the wavelength of the incident radiations as well as the nature of the scattering substance but depends only upon the angle of scattering. When = 900 , = = 0.02420A. This is known as the Compton wavelength of the electron and is denoted as . = effect or Compton shift is

= = (1-)

**Question :** Give the distinction between Compton wavelength and Compton shift.

**Ans.** Compton wavelength is a constant quantity = 0.02420A.

It is the Compton shift for an angle of scattering 900

Compton shift is a variable quantity depending upon the angle of scattering.

**Significance of Compton effect :** The Compton effect clearly shows the particle like nature of e.m radiation. Not only a precise quantum of energy h can be assigned to a photon but also a precise quantum of momentum = = . The total momentum of a monochromatic radiation cannot have any value but is only an exact multiple of the linear momentum of a single photon. In other words, the momentum as well as the energy of the e.m. radiation is quantized. The phenomenon of Compton effect is thus due to the elastic collision between two particles, the photon of the incident radiation and electron of the scatterer.

**Demonstration of Compton effect :**



A beam of monochromatic x-rays of known wave length is made incident on graphite scatterer. The scattered x-rays are received by a Bragg’s X-rays spectrometer and their wavelengths are determined at various angle. The intensity distribution versus wavelength for various scattering angles is plotted.



It is oblivious that the waves have two peaks, one corresponding to unmodified radiations and the other corresponding to modified radiations. The difference between two peaks on wavelength axis gives the Compton shift This is in agreement with the comption shift given by the formula.

= (1-)

When =900 the experimentally observed = 0.0242

This is in agreement with the Compton formula for = 900, ie, = = .02420

Thus Compton effect is experimentally demonstrated.

**Questions :**

1. Show that Compton shift is independent of the wavelength of incident x-rays.
2. How do you account for the presence of unmodified component of x-rays in the study of Compton effect ?.
3. Why Compton effect cannot be observed with visible light?

**Pair production** : Pair production is an example for materialization of energy. In this process, the photon disappears and is converted to an electron- positron pair ie, in this process e.m energy is converted into matter. This process can take place only when the photon energy exceeds 2moc2. This is the threshold energy for pair production. The pair production process cannot take place in free space and usually takes place in the presence of nuclear field.

The conversion of a photon into an electron and a positron ( ie. a + vely charged electron), when the photon traverses the strong electric field surrounding a nucleus is called pair production.

ie. = ++

= rest energy of electron

= rest energy of positron

= K.E of electron

= K.E of positron.

**Annihilation of matter:** The inverse of the materialization of energy is the annihilation of matter. When a positron combines with an electron both disappear producing two quanta of γ - rays. i.e, + γ + γ. This process is called pair annihilation.

**Photons and gravity:**

Photons behave as though they have gravitational mass. We know, the rest mass of photon is zero but it has inertial mass, called as photon mass

i.e, photo mass m = = = …………………(1) m = =

But gravitational mass is always equal to inertial mass. So a photon of frequency should act gravitationally like a particle of mass m =

**[ Note:** when a stone of mass m is dropped from a height H near the earth’s surface we have ½ = mgH . ∴ final speed v = ]

Consider a photon falling through a height H. The photon gains the energy mgH. This gain in energy is manifested as an increase in frequency from

Final photon energy = initial photon energy + increase in energy

= h + mgH m =

= h + gH

∴Photon energy after falling through a height H is

= h or, = (1+ ).

K.E = 0 E = h

K.E= mgH E= h+ gH =

**Gravitation red shift**

The gravitational behavior of light predicts an interesting astronomical effect ie, if the frequency of a photon moving towards the earth increases, then frequency of a photon moving away from it should decrease.

Suppose a photon of initial frequency is emitted by a star of mass M and radius R. The P:E of mass m on the surface of star is,

P.E = - ve sign shows force between M and m is attractive

P.E of a photon of mass on the star’s surface = x ie, m =

Quantum energy of the photon = h

∴ Total energy of the photon = P.E + its quantum energy.

i.e, E = + h

E = h

At a larger distance from the star, for instance at the earth, the photon is beyond the star’s gravitational field, but its total energy remains the same. The photon’s energy is now entirely electromagnetic. Let be the frequency of the arriving photon at the earth.

h or =

Gravitational red shift = relative frequency change = = = 1- =

The photon has a lower frequency at the earth corresponding to its loss in energy as it leaves the field of the star.

∴ a photon in the visible region of the spectrum is thus shifted towards the red end and this phenomenon is accordingly known as gravitational red shift. or

There is a red shift for the spectral lines of light emitted by an atom situated in a gravitational field when this light is observed in a gravitation free part of space. This is known as gravitational red shift.

**White dwarfs**: Gravitational red shift can be measured for a class of stars known as white dwarfs. A white dwarf is an old star whose interior consists of atoms where electron structures have collapsed and so it is very small; a typical white dwarf is about the size of the earth but has the mass of the sun.

No white dwarf star can have a mass larger than Mo = 1.4 x Mass of the sun. This mass is known as the Chandrasekhar limit.

**Quasars :** Quasar is the short form for quasi-stellar radio sources. A quasar is an object that emits enormous power of the order of 1053 W, which is equivalent of the energy of a million suns. Spectroscopic observations show that they move with very high speeds approaching 0.9 times the speed of light. It is believed that such an enormous power output of a quasar may be due to gravitational collapse.

Quasars appear as a sharp point of light. Quasars are powerful sources of radio waves. Hence their name, quasi-stellar radio source. Most astronomers believe that at the heart of every quasar is a black hole whose mass is at least that of 100 million suns.

**Black hole :** A star whose radius is smaller than its Schwarz child radius is known as a black hole. A black hole is a super dense planetary material formed due to the death of a star of mass more than five solar masses. Thus it has a very small size due to continued compression of the core of the star during explosion. As a result of it, the gravitational pull of a black hole is very strong. It is called a black hole for the reason that any particle or photon approaching its surface is just swallowed by it. On the other hand, no particle or photon of radiation emitted by its surface can escape it. Since radiation is neither emitted nor reflected by it, it appears black and is called a black hole.

**Schwarz child radius Rs:** The Schwarz child radius of a body of mass M is defined as

Rs =

According to general theory of relativity, the correct criterion for a star to be black hole is

= ≥ ½

ie if an object of mass M has a radius Rs = the escape velocity would be c. This expression was derived by Schwarz child. ∴ Rs is called Schwarz child radius. A star whose radius is smaller than Schwarz child radius is known as a black hole.