N - N scattering

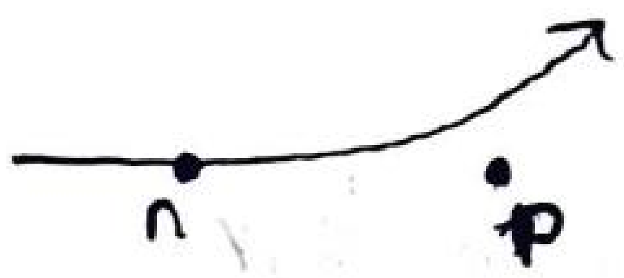
* Y? — To reveal p-p , p-n, n-n interaction.

* for a low energy system, the schroodinger eqn. in polar co-ordinates.

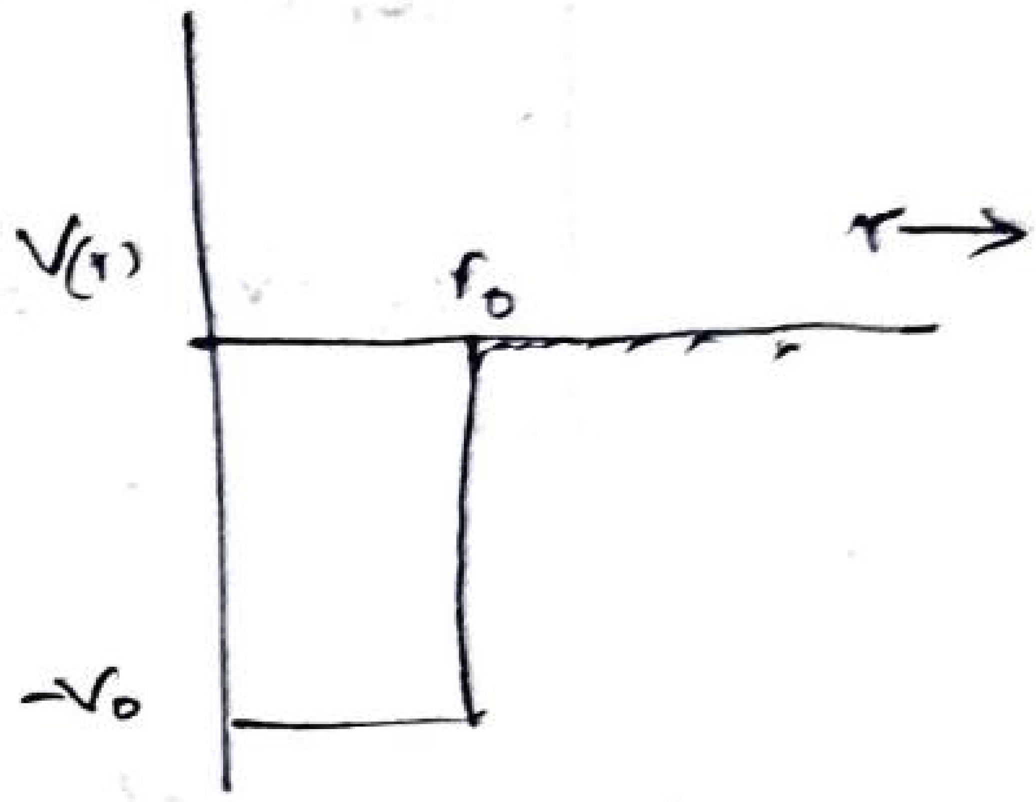
$$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2(\psi)}{\partial r^2} + \frac{2}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \psi}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 \psi}{\partial \phi^2}\right) + V\psi \right] = i\hbar \frac{\partial \psi}{\partial t}.$$

(V - nuclear potential.)

* Scattering $\longrightarrow$



assume a spherical squarewell potential.



varrable separation,

$$\psi = \underline{R(r)}\ \underline{\Theta(\theta)}\ \underline{\Phi(\phi)}\ \underline{e^{-\frac{iEt}{\hbar}}}$$

↳ time also separated from space co-ordinates.

↳ The radial wave function, $R(r) = \frac{u(r)}{r}$ satisfies the eqn.

$$\boxed{-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \left[V(r) + \frac{l(l+1)\hbar^2}{2mr^2}\right] u = E u}$$

where 'l' is the orbital angular momentum quantum no.

↳ We have to concentrate 'l' equal to particular value here we stick to $\underline{\underline{l=0}}$ then S/n. to the above eqn.

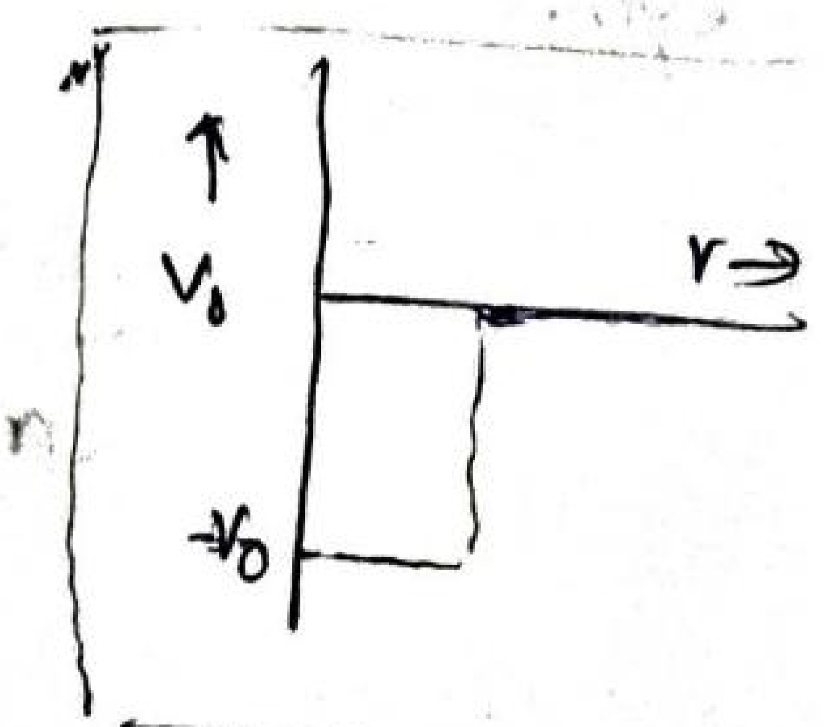
$$\boxed{\begin{aligned} u_1(r) &= A \sin k_1 r + B \cos k_1 r && \text{— } r < r_0 \\ u_2(r) &= C_2 \sin k_2 r + D_2 \cos k_2 r && \text{— } r > r_0 \end{aligned}}$$

↳ Looking at the schrod. eqn. we have $V = V_0$ up to $r = r_0$.

So for that part energy to be larger than the potential.

↳ Where $\boxed{k_1 = \sqrt{\frac{2m(E+V_0)}{\hbar^2}} \quad ; \quad k_2 = \sqrt{\frac{2mE}{\hbar^2}}}$

For the wave function to be finite as $r \to 0$



∴ wave function.

$$\psi(r) = R(r) = \frac{u(r)}{r} = \frac{A \sin k_1 r}{r} + \frac{B \cos k_1 r}{r}.$$

∴ we have for wave fun. to be finate as $r \to 0$.

$$\therefore \lim_{r \to 0} \frac{\sin k_1 r}{r} = k_1 \quad \& \quad \lim_{r \to 0} \frac{\cos k_1 r}{r} = \infty$$

(So. we cannot take 2nd part as soln.)

↳ Applying boundary conditions

$$\boxed{1.\ u_1(r_0) = u_2(r_0)} \quad 2.\ \left.\frac{\partial u_1}{\partial r}\right|_{r=0} = \left.\frac{\partial u_2}{\partial r}\right|_{r=0}.$$

$\therefore C_2 \sin k_2 r_0 + D_2 \cos k_2 r_0 = A \sin k_1 r_0$

taking derivative

$$k_2 C_2 \cos(k_2 r_0) + - k_2 D_2 \sin(k_2 r_0)$$

$$= k_1 A \cos k_1 r_0 .$$

Redefining constants to make the sln. better form.

$$- C_2 = C \cos \delta_0 \quad , D_2 = C \sin \delta_0 .$$

boundary conditions to be

$$C \sin(k_2 r_0 + \delta_0) = A \sin(k_1 r_0)$$

and

$$k_2 C \cos(k_2 r_0 + \delta_0) = k_1 A \cos(k_1 r_0)$$

$k/2$ Dividing 2nd one by 1st one

we get $\underline{k_2 \cot(k_2 r_0 + \delta_0) = k_1 \cot(k_1 r_0)}$

we can find out δ_0 from known

k_1 & k_2.

x ——————————————— x.

We have <u>incident wave func.</u>

$$\psi_{in} = A \sin kr \; e^{-i\omega t}.$$

r - impact parameter

we can rewrite eqn. in terms of exponentials.

$$\boxed{\psi_{in} = \frac{A}{2ik}\left(\frac{e^{ikr}}{r} - \frac{e^{-ikr}}{r}\right) e^{-i\omega t}}$$

where in this eqn.

$\frac{e^{-ikr}}{r}$ is incoming spherical wave

$\frac{e^{ikr}}{r}$ is outgoing spherical wave

After the scattering

$$\boxed{\psi_{(r)} = \frac{A}{2ik}\left(\frac{e^{i(kr+\beta)}}{r} - \frac{e^{-ikr}}{r}\right)}$$

(small change in outgoing part after scattering :))

we are writing the previous eqn. for $\psi(r)$

$$\psi(r) = \frac{C}{r} \sin(k_2 r + \delta_0)$$

$$= \frac{C}{2i}\left[\frac{e^{i(k_2 r + \delta_0)}}{r} - \frac{e^{-i(k_2 r + \delta_0)}}{r}\right]$$

$$\psi_{(r)} = \frac{C}{2i}\left(\frac{e^{i(k_2 r + 2\delta_0)}}{r} - \frac{e^{-ik_2 r}}{r}\right) e^{-i\delta_0}$$

Compare this eqn with

$$\psi = \frac{A}{2ik}\left(\frac{e^{ikr+\beta)}}{r} - \frac{e^{-ikr}}{r}\right)$$

we have, $\beta = 2\delta_0$ & $A = kCe^{-i\delta_0}$:

we can again write, $\psi_{in} = \frac{A}{2ik}\left(\frac{e^{ikr}}{r} - \frac{e^{-ikr}}{r}\right)$

after scattering $\psi = \frac{A}{2ik} e^{-i\delta_0}\left(\frac{e^{i(kr+\beta)}}{r} - \frac{e^{-ikr}}{r}\right)$

$\therefore$ scattered wave fun. $\boxed{\psi_{sc} = \psi - \psi_{in} = \frac{A}{2ik}\left(e^{i2\delta_0} - 1\right)\frac{e^{ikr}}{r}}$.

* We have total cross section,

$$\boxed{\sigma = \frac{4\pi \sin^2 \delta_0}{k^2}}$$

we have considered $l=0$. ~~so~~ ~~we~~ which is actually spherical symmetric situation.

↳ assume that incident energy is small, say

$$E \lesssim 10\,keV.$$

from studies of Deuteron States, we have

$$V_0 = 35\ MeV$$

~~as~~ we can apply these in eqn. for k_1 & k_2. then

$$k_1 = \sqrt{2m(E+V_0)/\hbar^2} = 0.92\ fm^{-1}.$$

$$k_2 = \sqrt{2mE/\hbar^2} = 0.016\ fm^{-1}$$

↳ we have, $k_2 \cot(k_2 R + \delta) = k_1 \cot k_1 R.$

here right side of this equation is taken as $-\alpha$

$$\therefore \boxed{\alpha = -k_1 \cot k_1 R.}$$

by trignometric manipulations, we can write

$$\sin \delta_0 = \frac{\cos k_2 R + (\alpha/k_2) \sin k_2 R}{1+(\alpha^2/k_2^2)}$$

$$\therefore \quad \sigma = \frac{4\pi}{(k_2^2 + \alpha^2)} \left(\cos k_2 R + \frac{\alpha}{k_2} \sin k_2 R \right)$$

from the study of Deuterium isotope ^2H,

we have $R = 2$.

$\alpha = 0.2\ fm^{-1}$

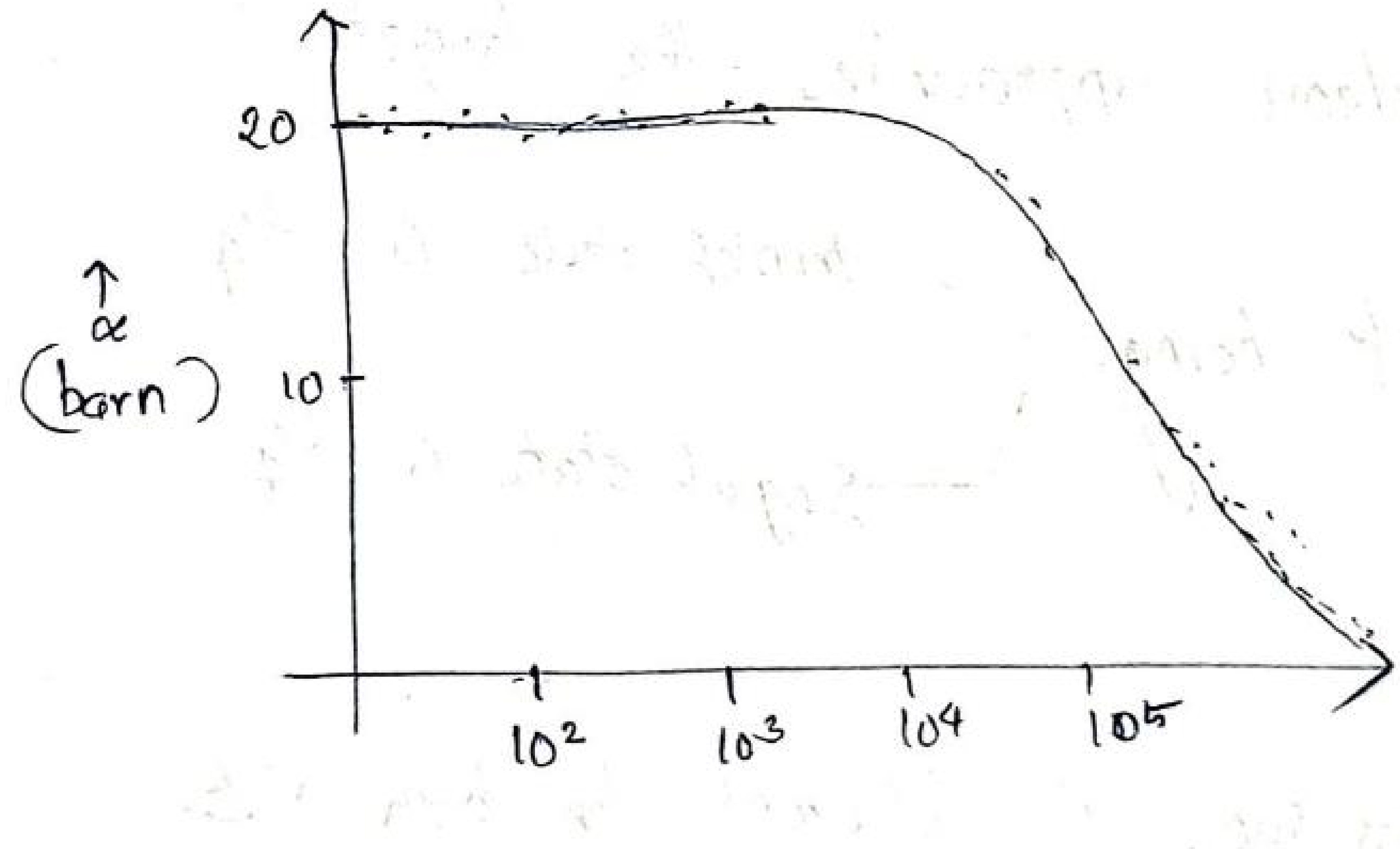
$\therefore k_2^2 << \alpha^2$ & $k_2 R << 1$ giving

$$\sigma = \frac{4\pi}{(\alpha^2)} (1 + \alpha R) = 4.6\ b. \quad \text{(In the range of 4–5 b)}$$

where 1 barn $= 10^{-28}\ m^2$.

this suggest that σ should be constant at low energy.

↳ as per experimental cross section data of neutron by proton



from graph we are getting

1.) Cross section is constant at low energy.

2.) but low energy cross sectn. 20.4 barn.

not in agreement with calculated value of 4–5 barn spin of incident and scattered nucleon.

- the proton and neutron spins $(\frac{1}{2})$ can combine to give total spin $\boxed{S = s_p + s_n}$

(can have value either 0 or 1)

- for $S = 1$ — has 3 orientations corres. to 3 components $-1, 0, +1$

for $S = 0$ — has only single orientation.

$\therefore$ $S = 1$ is called triplet state.

$S = 0$ is called singlet state.

$\therefore$ as the ~~neutr~~ nucleon approaches the target

the probability of being — triplet state is $\frac{3}{4}$
— singlet state is $\frac{1}{4}$

if scattering cross sec. is different for both states

$$\therefore \sigma = \frac{3}{4}\sigma_1 + \frac{1}{4}\sigma_2$$

σ_1, σ_2 — cross sec. for scattering in triplet & singlet states.

$\therefore$ by calculating eqn. $\alpha = \frac{4\pi}{\alpha^2}(1+\alpha R)$, we used parameters obtained from deuteron, which is in $S=1$ state.

$\therefore$ $\sigma_1 = 4.6\,b$.

by using measured value $\sigma = 20.4\,b$.

$\therefore$ for low energy $\sigma \Rightarrow \underline{\sigma_s = 67.8\,b}$.

This calculation shows that there is large difference between 'σ' in singlet & triplet states. ie. NUCLEAR FORCE MUST BE SPIN DEPENDENT.