Q The map $\phi : Z \longrightarrow Z$ defined by $\phi(n) = n+1$ for $n \in Z$. is one-to one and onto Z. Give the definition of the binary operation $*$ on Z s. ϕ is an isomorphism mapping.

(a) $(Z, \cdot)$ onto $(Z, *)$

(b) $(Z, *)$ onto $(Z, \cdot)$

An:- since ϕ is an isomorphism mapping $(Z, \cdot)$ onto $(Z, *)$

for any $n, m \in Z$

$$\phi(n \cdot m) = \phi(n) * \phi(m) \quad \text{——— ①}$$

we have $\phi(n) = n+1$

$$\therefore n = \phi(n-1)$$

$$m = \phi(m-1) \quad \text{——— ②}$$

$$\begin{aligned} \therefore n * m &= \phi(n-1) * \phi(m-1) \\ &= \phi((n-1) \cdot (m-1)) \\ &= \phi(nm - n - m + 1) \\ &= nm - n - m + 1 + 1 \\ &= nm - n - m + 2. \end{aligned}$$

$\therefore \boxed{n*m = nm - n - m + 2}$

Now we want to find the identity element

$n*x = n = x*n \quad [n \in Z, x \in Z]$

we want to find the value of x.

~~$n*x = nx - n - x + 2.$~~

$n*x = n$

$\Rightarrow nx - n - x + 2 = n.$

$\Rightarrow nx - n - x + 2 - n = 0.$

$\Rightarrow nx - x + 2 - 2n = 0.$

$\Rightarrow n(x-2) - (x-2) = 0.$

$\Rightarrow (n-1)(x-2) = 0$

$\Rightarrow x = 2.$

$x*n = n.$

$\Rightarrow xn - x - n + 2 = n$

$\Rightarrow n(x-2) - (x-2) = 0$

$\Rightarrow (n-1)(x-2) = 0.$

$\Rightarrow x = 2.$

$\therefore n*2 = n = 2*n \quad \forall n \in Z$ and $2 \in Z$

Hence 2 is an identity element $*$ on Z