Polar Form

Let 'r' and 'θ' be the polar co-ordinates of the point (x, y) which corresponds to a non-zero complex number x + iy. Since

$$x = r\cos\theta \text{ and}$$
$$y = r\sin\theta.$$

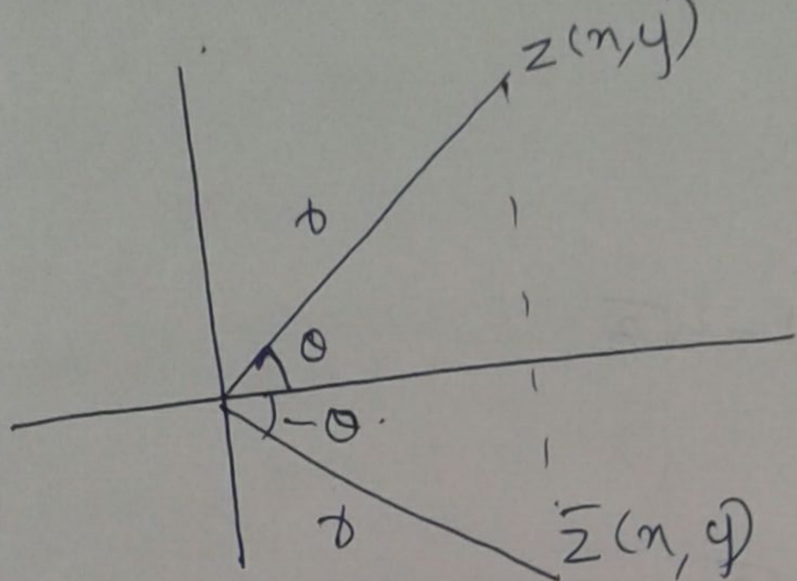
we have

$$z = x + iy$$
$$= r\cos\theta + i r\sin\theta$$
$$= r(\cos\theta + i\sin\theta)$$

$$\boxed{\therefore\ z = r(\cos\theta + i\sin\theta)}$$

Here $r = \sqrt{x^2+y^2} = |z|$

the angle $\theta = \tan^{-1}\left[\frac{y}{x}\right]$ is called argument or amplitude of z and is denoted by arg z or amp z.



The unique value of θ that lies in the interval $-\pi < \theta \leq \pi$ is called the principal value of arg z and is denoted by Arg z.

* $\arg z = \operatorname{Arg} z + 2n\pi$ $[n = 0, \pm 1 \ldots]$
* when z is a negative real number $\operatorname{Arg} z = \pi$

Q Find the polar Form of the following complex numbers.

(I) $1-i\sqrt{3}$

(II) $-2+2i\sqrt{3}$

(III) $-\sqrt{3}-i$

An:-

(I) given

$$z = 1-i\sqrt{3}$$

$$\therefore x = 1$$
$$y = -\sqrt{3}$$

$(1, -\sqrt{3})$ is the given point.

now $r = |z|$
$$= \sqrt{x^2+y^2}$$
$$= \sqrt{1^2+3}$$
$$= \sqrt{4}$$
$$= 2.$$

now we want to find θ or $\arg z$.

now $\tan^{-1}\left|\frac{y}{x}\right| = \theta$

$$\therefore \theta = \tan^{-1}\left|\frac{-\sqrt{3}}{1}\right|$$
$$= \tan^{-1}(\sqrt{3})$$
$$= \pi/3$$

now we want to find $\operatorname{Arg} z$.

$z = (1, -\sqrt{3})$ is in the Fourth quadrant.

$\operatorname{Arg} z = -\alpha$.

$\therefore \operatorname{Arg} z = -\frac{\pi}{3}$

now $z = r[\cos\theta + i\sin\theta]$

$= 2\left[\cos\frac{-\pi}{3} + i\sin\frac{\pi}{3}\right]$