

Powers and Roots

Exponential Form

we have $e^{j\theta} = \cos\theta + j\sin\theta$.

then $z = r(\cos\theta + j\sin\theta)$

$$= r e^{j\theta}$$

powers and roots

$$z = r e^{i\theta}$$

$$\text{Then } z^n = r^n e^{in\theta}$$

$$= r^n (\cos n\theta + i \sin n\theta) \quad n = 0, \pm 1, \pm 2, \dots$$

n th roots of a non zero complex number

$$c_k = \sqrt[n]{r_0} \left[\cos \left[\frac{\theta_0 + 2k\pi}{n} \right] + i \sin \left[\frac{\theta_0 + 2k\pi}{n} \right] \right]$$

$k = 0, 1, \dots, n-1$

Q Find the 3 cube roots of $-8i$

an: - The polar form of $-8i$ is

$$-8i = 8 \left[\cos \left[-\frac{\pi}{2} \right] + i \sin \left[-\frac{\pi}{2} \right] \right]$$

$$= 8 \left[\cos \left[-\frac{\pi}{2} + 2k\pi \right] + i \sin \left[-\frac{\pi}{2} + 2k\pi \right] \right]$$

$$\left[k = 0, \pm 1, \pm 2 \right]$$

$$= 2 \left[\cos \left[\frac{-\pi}{6} + \frac{2k\pi}{3} \right] + i \sin \left[\frac{-\pi}{6} + \frac{2k\pi}{3} \right] \right]$$

($k=0, 1, 2$)

Hence the 3 cube root of $-8i$ are

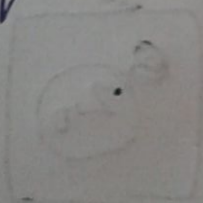
$$c_0 = \sqrt{3} - i$$

$$c_1 = 2i$$

$$c_2 = -\sqrt{3} - i$$

H.W

① Find the 2 square roots of $\sqrt{3} + i$



$$\begin{aligned} \therefore (-8i)^{1/3} &= \left[8 \left[\cos\left(-\frac{\pi}{2} + 2k\pi\right) + i\sin\left(-\frac{\pi}{2} + 2k\pi\right) \right] \right]^{1/3} \\ &= 2 \left[\cos\left[-\frac{\pi}{6} + \frac{2k\pi}{3}\right] + i\sin\left[-\frac{\pi}{6} + \frac{2k\pi}{3}\right] \right] \quad (k=0,1,2) \end{aligned}$$

Hence the 3 cube root of $-8i$ are

$$c_0 = \sqrt{3} - i$$

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① Find the 2 square roots of $\sqrt{3} + i$

