Powers and Roots

Exponential Form

we have $e^{i\theta} = \cos\theta + i\sin\theta$.

then $z = r(\cos\theta + i\sin\theta)$

$= re^{i\theta}$

powers and roots

$$z = re^{i\theta}$$

Then $z^n = r^n e^{in\theta}$

$$= r^n(\cos n\theta + i\sin n\theta) \quad n = 0, \pm 1, \pm 2 \ldots$$

n^{th} roots of a non zero complex number

$$c_k = \sqrt[n]{r_0}\left[\cos\left(\frac{\theta_0 + 2k\pi}{n}\right) + i\sin\left(\frac{\theta_0 + 2k\pi}{n}\right)\right]$$

$$k = 0, 1 \ldots n-1$$

Q Find the 3 cube roots of $-8i$

an:- The polar form of $-8i$ is

$$-8i = 8\left[\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right]$$

$$= 8\left[\cos\left[-\frac{\pi}{2} + 2k\pi\right] + i\sin\left(-\frac{\pi}{2} + 2k\pi\right)\right]$$

$$\left(k = 0, \pm 1, \pm 2 \ldots\right)$$

$$= 2\left[\cos\left(\frac{-\pi}{6}+\frac{2k\pi}{3}\right)+i\sin\left[\frac{-\pi}{6}+\frac{2k\pi}{3}\right]\right]$$

$(k = 0, 1$

Hence the 3 cube root of $-8i$ are

$c_0 = \sqrt{3} - i$

$c_1 = 2i$

$c_2 = -\sqrt{3} - i$

H.W

(1) Find the 2 square roots of $\sqrt{3} + i$

$$\therefore (-8i)^{1/3} = \left(8\left[\cos\left(\frac{-\pi}{2}+2k\pi\right)+i\sin\left[\frac{-\pi}{2}+2k\pi\right]\right]\right)^{1/3}$$

$$= 2\left[\cos\left[\frac{-\pi}{6}+\frac{2k\pi}{3}\right]+i\sin\left[\frac{-\pi}{6}+\frac{2k\pi}{3}\right]\right]$$

$(k=0,1,2)$

Hence the 3 cube root of $-8i$ are

$c_0 = \sqrt{3}-i$

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H.W

(1) Find the 2 square roots of $\sqrt{3}+i$