Regions in the complex plane

Neighbourhood

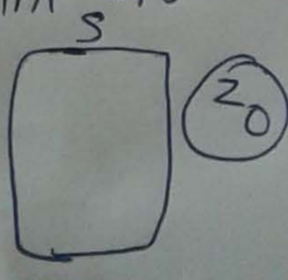
Let 'z_0' be a point in the complex plane. Then the neighbourhood of the point z_0 is defined as the set of all points for which $|z-z_0|<\varepsilon$. where ε is an arbitrary small positive number, usually called the radius of this neighbourhood.

Exterior, interior and Boundary points

Let 'S' be a non empty subset of the complex plane.

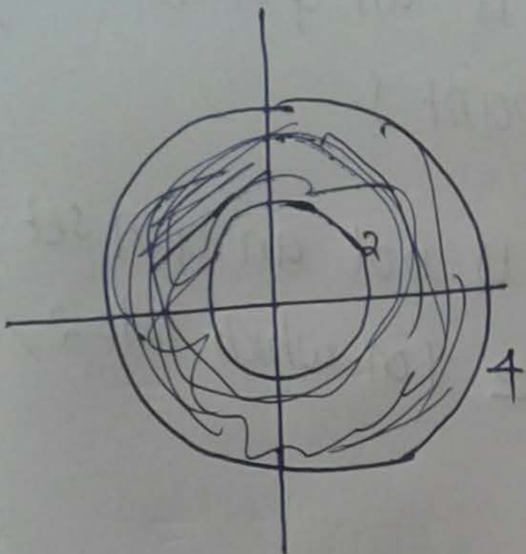
A point 'z_0' is said to be an interior point of the set S. If $\exists$ a neighbourhood with centre z_0 that contains only points of S.

A point 'z_0' is said be an exterior point of the set S. If $\exists$ a neighbourhood with centre 'z_0' which contains no point of S.



If 'z_0' is neither an interior point nor an exterior point it is called Boundary point.

Eg: let $S = \{z : 2 < |z| < 4\}$



S is the region between two concentric circles with center origin and radius 2 and 4. Such a region is called a circular ring or annulus.

* Every point between the 2 concentric circles $|z|=2$ and $|z|=4$ are interior point of S.

* Every point within the circle $|z|=2$ and outside the circle $|z|=4$ are exterior point of S.

* Every point on the circles $|z|=2$ and $|z|=4$ is a boundary point of S.

Open set

If every point a set is an interior point, then it is called an openset, in otherwords, a set is open if it contains none of its boundary points.

Eg:- $S = \{2 < |z| < 4\}$ is an open set every point of S is an interior point S.

$S_1 = \{z : 2 \leq |z| < 4\}$ is not an open set since it contains the points z for which $|z| = 2$, the boundary points of S_1.

Limit point or Accumulation point

Let S be a non empty sub set of the complex plane. A point z_0 is a ***limit point*** or an ***accumulation point*** of S, if every neighbourhood of z_0 however small, contains at least one point of S other than z_0.

Note that limit points are either interior points or boundary points of a set and every interior point is a limit point of the set.

Example. Let $S = \left\{ i^n/n \,:\, n = 1,\ 2,\ 3, \ldots \right\}$. Then the only limit point of is zero.

Closed set

A set is ***closed*** if it contains all its limit points. In other words, a set is osed if it contains all of its boundary points.

Example. $S = \{z : 2 \leq |z| \leq 4\}$ is a closed set.

Some sets are, of course, neither open nor closed. For a set to be not pen, there must be a boundary point that is contained in the set and for a set be not closed there exists a boundary point not contained in the set. bserve that the punctured disk $0 < |z| \leq 1$ is neither open nor closed. The t of all complex numbers is, on the other hand, both open and closed since has no boundary points.

Closure of a set

The closure $\overline{S}$ of a set S is the closed set consisting of all points in S gether with the boundary of S.

Example. The closure of the annulus $2 < |z| < 4$ is $2 \leq |z| \leq 4$.

Connected set

If any two points of a given set S an be joined by a polygonal line, onsisting of a finite number of line egments joined end to end, such that e poligonal line lies completely within , then the set S is said to be a *onnected set*.

Example. $S = \{z : 1 \leq |z| \leq 3\}$ is connected set. (refer figure).

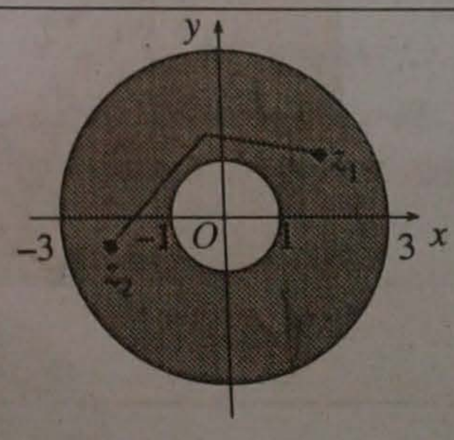


FIGURE 5.10

Domain

A connected open subset of the complex plane is called a ***domain***.

Example. The annulus $1 < |z| < 2$ is a domain.

Bounded set

A set S is ***bounded*** if every point of S lies inside some circle $|z| = r$,