$x = 0,\ u = \log_{10} 1 = 0$ and $x = a,\ u = \log_{10}$

$$\int_0^a \frac{2\log(x+1)}{x+1}\,dx = \int_0^1 2u \cdot \ln 10\, du$$

$$= \ln 10 \int_0^1 2u\, du$$

$$= \ln 10\, [u^2]_0^1$$

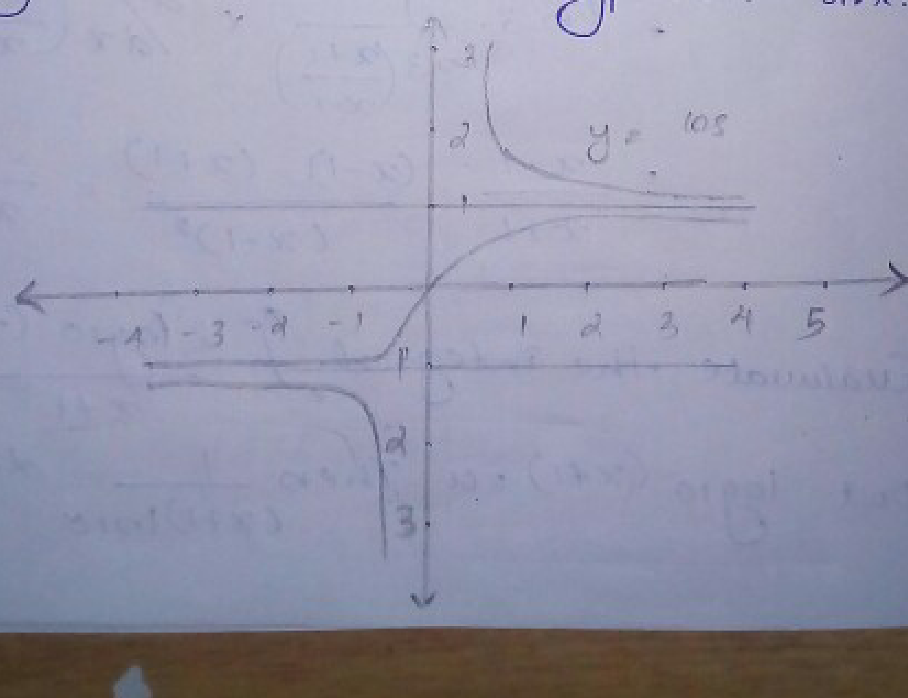
$$= \ln 10.$$

→ Base 10 logarithms.

Base 10 logarithms are commonly called common logarithms.

→ hyperbolic functions.

Definition :- The even and odd parts in the representation ~~are~~ of e^x are called the hyperbolic cosine and hyperbolic sine.



Using the definition we can define.

hyperbolic Cosine of x $\quad \cosh x = \frac{e^x + e^{-x}}{2}$

Sine of x $\quad \sinh x = \frac{e^x - e^{-x}}{2}$

hyperbolic tangent

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

hyperbolic cotangent

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

hyperbolic Secant $\quad \operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$

hyperbolic cosecant $\quad \operatorname{cosech} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$

→ identities in hyperbolic functions.

1. $\sinh(-x) = -\sinh x$
2. $\cosh(-x) = \cosh x$
3. $\sinh 0 = 0$
4. $\cosh 0 = 1$
5. $\cosh 2x = \cosh^2 x + \sinh^2 x = 1 + 2\sinh^2 x$
   $= 2\cosh^2 x - 1$
6. $\sinh 2x = 2\sinh x \cosh x$
7. $\sinh x = 2\sinh x \cosh x$

Using the definition we can define.

hyperbolic cosine of x $\cosh x = \frac{e^x + e^{-x}}{2}$

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1. $\sinh(-x) = -\sinh x$
2. $\cosh(-x) = \cosh x$
3. $\sin 0 = 0$
4. $\cos 0 = 1$
5. $\cosh 2x = \cosh^2 x + \sinh^2 x = 1 + 2\sinh^2 x$

$$= 2\cosh^2 x - 1$$

6. $\sinh 2x = 2\sinh x \cosh x$
7. $\sinh x = 2\sinh x \cosh x$

8. $\cosh^2 x - \sinh^2 x = 1$

9. $\tanh^2 x + \operatorname{sech}^2 x = 1$

10. $\coth^2 x - \operatorname{cosech}^2 x = 1$

11. $\cosh^2 x = \dfrac{\cosh 2x + 1}{2}$

12. $\sinh^2 x = \dfrac{\cosh 2x - 1}{2}$

13. $\cosh 3x = 4\cosh^3 x - 3\cosh x$

14. $\sinh 3x = 3\sinh x + 4\sinh^3 x$

15. $\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$

16. $\cosh(x-y) = \cosh x \cosh y - \sinh x \sinh y$

17. $\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$

18. $\sinh(x-y) = \sinh x \cosh y - \cosh x \sinh y$

19. $\tan(x+y) = \dfrac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$

20. $\tan(x-y) = \dfrac{\tanh x - \tanh y}{1 - \tanh x \tanh y}$

21. $\sinh 2x = \dfrac{2\tanh x}{1 - \tanh^2 x}$

22. $\cosh 2x = \dfrac{1 + \tanh^2 x}{1 - \tanh^2 x}$

23. $\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$

24. $\tanh 3x = \dfrac{3\tanh x + \tanh^3 x}{1 + 3\tanh^2 x}$

8. $\cosh^2 x - \sinh^2 x = \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left[\frac{e^x - e^{-x}}{2}\right]^2$

$$= \left(\frac{e^{2x} + e^{-2x} + 2}{4}\right) - \left(\frac{e^{2x} + e^{-2x} - 2}{4}\right)$$

$\cosh^2 x - \sinh^2 x = 1$

9. dividing both sides of ① by $\cosh^2 x$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\tanh^2 x + \operatorname{sech}^2 x = 1$$

10. Dividing both sides of equation (2) by $\sinh^2 x$ we get

$$\coth^2 x - \operatorname{cosech}^2 x = 1$$

Example 1 :- prove that $\cosh^2 x/2 = 1/2 (\cosh x + 1)$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\frac{\cosh x + 1}{2} = \frac{\frac{e^x + e^{-x}}{2}}{2} = 1/4 \left(e^x + e^{-x} + 2\right)$$

$$= 1/4 \left(e^x + e^{-x} + 2 e^x e^{-x}\right)$$

Since $e^x e^{-x} = 1$

$$= \left(\frac{e^{x/2} + e^{-x/2}}{2}\right) = \left(\cosh x/2\right)^2$$

$$= \cosh^2 x/2$$

In view of the above example

$$\cosh x/2 = \sqrt{\frac{\cosh x + 1}{2}}$$

(1) $\cosh^2 x - \sinh^2 x = 1$

$\rightarrow \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$

$$= \frac{e^{2x} + e^{-2x} + 2e^x.e^{-x} - (e^{2x} + e^{-2x} - 2e^x.e^{-x})}{4}$$

$$= \frac{\cancel{e^{2x}} + \cancel{e^{-2x}} + 2e^0 - \cancel{e^{2x}} - \cancel{e^{-2x}} + 2.e^0}{4}$$

$$= \frac{4}{4} = 1$$

(2) $d/dx (\sinh x) = d/dx \left(\frac{e^x - e^{-x}}{2}\right)$

$$= 1/2 [e^x - e^{-x} \cancel{x} - 1]$$

$$= 1/2 (e^x + e^{-x})$$

$$= \cosh x$$

(3) $d/dx (\cosh x) = d/dx \frac{(e^x + e^{-x})}{2}$

$$= 1/2 (e^x + e^{-x} \cancel{x} - 1)$$

$$= \frac{e^x - e^{-x}}{2}$$

$\frac{d}{dx}(\cosh x) = \sinh x.$

(4) $\frac{d}{dx}(\tanh x) = \frac{d}{dx}\left(\frac{\sinh x}{\cosh x}\right)$

$$= \frac{\cosh x \cdot \frac{d}{dx}(\sinh x) - \sinh x \frac{d}{dx}(\cosh x)}{\cosh^2 x}$$

$$= \frac{\cosh x \cdot \cosh x - \sinh x \sinh x}{\cosh^2 x}$$

$$= \frac{1}{\cosh^2 x} = \left(\frac{1}{\cosh x}\right)^2$$

$$= \underline{\underline{\operatorname{sech}^2 x}}$$

(5) $\frac{d}{dx}(\coth x)$ (6) $\frac{d}{dx}(\operatorname{sech} x)$

$\rightarrow$

7. Differentiate the following function with respect to x.

(i) $x \sinh x - \cosh x$

$\rightarrow$ $\frac{d}{dx}(x \sinh x - \cosh x)$

$= x \cosh x + \sinh x - \sinh x$

$= x \cosh x$

(ii) $\frac{d}{dx}(\tanh \sqrt{1+x^2})$

$\rightarrow$ $= \operatorname{sech}^2 \sqrt{1+x^2} \cdot \frac{d}{dx}(\sqrt{1+x^2})$

$= \operatorname{sech}^2 \sqrt{1+x^2} \cdot \frac{1}{2\sqrt{1+x^2}} \times 2x$

$= \frac{x}{\sqrt{1+x^2}} \operatorname{sech}^2 \sqrt{1+x}$.

(iii) $\frac{d}{dx}(\ln|\tanh x/2|)$

$= \frac{1}{\tanh(x/2)} \times \frac{d}{dx}(\tanh x/2)$

$= \frac{1}{\tanh(x/2)} \operatorname{sech}^2 x/2 \times 1/2$

$= \frac{1}{2\sinh(x/2)\cosh(x/2)}$

$= \frac{1}{\sinh 2x/2} = \frac{1}{\sinh x} = \operatorname{cosech} x$

$$\frac{1}{\cosh x} = \operatorname{sech} x$$

$$\frac{1}{2} \frac{\cosh x/2}{\sinh x/2} \times \frac{1}{\cosh^2 x/2}$$

$$= \frac{1}{2} \frac{1}{\sinh x/2 \, \cosh x/2}$$

$$= \frac{1}{2 \sinh x/2 \cdot \cosh x/2}$$

→ hyperbolic identities.

$$\cosh^2 x - \sinh^2 x = 1$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\cosh x = \frac{1}{\sinh x}$$

$$\cot h x = \frac{1}{\tanh x}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

(1) Given $\sinh x = -3/4$ find other five hyperbolic function.

→ $$\cosh^2 x = 1 + \sinh^2 x$$

$$= 1 + 9/16$$

$$= 25/16$$

$\Rightarrow \cosh x = 5/4$

$\therefore \tanh x = \dfrac{\sinh x}{\cosh x}$

$= \dfrac{-3/4}{5/4}$

$= -3/5$

$\coth x = \dfrac{1}{\tanh x}$

$= \dfrac{1}{-3/5}$

$\operatorname{sech} x = \dfrac{1}{\cosh x} = 4/5$

$\operatorname{cosech} x = \dfrac{1}{\sinh x} = 1/-3/4 = -4/3$

(2) ST $(\cosh x + \sinh x)^n = \cosh x + \sinh x$

$\rightarrow$ $\cosh x = \dfrac{e^x + e^{-x}}{2}$

$\sinh x = \dfrac{e^x - e^{-x}}{2}$

$\cosh x + \sinh x = \dfrac{e^x + \cancel{e^{-x}}}{2} + \dfrac{e^x - \cancel{e^{-x}}}{2}$

$= \dfrac{2e^x}{2} = e^x$

$$= \frac{2e^x}{2} = e^x$$

$$(\cosh x + \sinh x)^n = (e^x)^n = e^{nx} \text{ ——— (1)}$$

$$\cosh nx = \frac{e^{nx} + e^{-nx}}{2}$$

$$\sinh nx = \frac{e^{nx} - e^{-nx}}{2}$$

$$\cosh nx + \sinh nx = \frac{e^{nx} + \cancel{e^{-nx}}}{2} + \frac{e^{nx} - \cancel{e^{-nx}}}{2}$$

$$= \frac{2e^{nx}}{2} = e^{nx} \text{ ——— (2)}$$

From (1) & (2) it follows that

$$(\cosh x + \sinh x)^n = \cosh nx + \sinh nx$$

(8) Given $\sinh x = -1/3$ find other 5 hyperbolic functions.

→ $\sinh x = -1/3$

$$\cosh x = \frac{1}{-1/3}$$

$$= -3$$

$$\tanh x = \frac{-1/3}{-3}$$

$$= -1/3$$

$$\coth x = \frac{1}{\tanh x}$$

$$= \frac{1}{-1/3}$$

$$\text{Cosech}\, x = \frac{1}{\sinh x}$$

$$= \frac{1}{+1/3}$$

$$\text{Sech}\, x = -1/3$$

Q. Differentiate the following function w.r.to x.

(i) $x \sinh x - \cosh x$

(ii) $\tanh\sqrt{1+x^2}$

(iii) $\ln|\tanh x|$

→ (i) $\frac{d}{dx}(x\sinh x - \cosh x) = \frac{d}{dx}(x\sinh x) - \frac{d}{dx}(\cosh x)$

$= x.\cosh x + \sinh x - \sinh x$

$= x\cosh x$

(ii) $\tanh\sqrt{1+x^2} = \text{sech}^2\sqrt{1+x^2} \times \frac{d}{dx}\sqrt{1+x^2}$

$= \text{sech}^2\sqrt{1+x^2} \cdot \frac{1}{2\sqrt{1+x^2}} \times 2x$

$= \frac{x}{\sqrt{1+x^2}} \cdot \text{sech}^2\sqrt{1+x^2}$

Q Evaluate the following Integrals.

(i) $\int \tanh 5x\,dx$ (ii) $\int \frac{\sinh x}{1+\cosh^2 x}\,dx$

(iii) $\int_0^{\ln 2} 4e^x \sinh x\,dx$ (iv) $\int_0^{\pi/2} 2\sinh(\sin\theta)\cos\theta\,d\theta$

→ (i) $\int \tanh 5x\,dx = \int \frac{\sinh 5x}{\cosh 5x}\,dx$

put $u = \cosh 5x$.

$du = 5\sinh 5x\,dx$

$\int \tanh 5x\,dx = \frac{1}{5}\int \frac{du}{u}$

$= \frac{1}{5}\ln|u| + c$

$= \frac{1}{5}\ln|\cos 5x| + c$

(ii) $\cosh x = u$

$x\,dx = du$

$\int \frac{\sinh x}{1+\cosh^2 x}\,dx = \int \frac{du}{1+u^2}$

$= \tan^{-1} u + c$

$= \tan^{-1}(\cosh x) + c$

(iii) $\int_0^{\ln 2} 4e^x \sinh x\,dx = \int_0^{\ln 2} 4e^x \frac{e^x - e^{-x}}{2}\,dx$

$= \int_0^{\ln 2} (2e^{2x} - 2)\,dx$

$$= \left[e^{2x} - 2x\right]_0^{\ln 2}$$

$$= e^{2\ln 2} - 2\ln 2 - 1$$

$$= 4 - 2\ln 2 - 1 = 3 - 2\ln 2$$

(iii) put $\sin\theta = u$

$$\cos\theta \, d\theta = du$$

$$\theta = 0 \, , \, u = 0$$

$$\& \text{ when } \pi/2 , u = 1$$

→ Graphs of hyperbolic functions.

Graph of $y = \sinh x$

we have $\sinh x = \dfrac{e^x - e^{-x}}{2}$

$$\sinh x = \frac{e^0 - e^{-0}}{2}$$

$$= \frac{1-1}{2} = 0$$

hence the curve $\sinh x$ passes through the origin

Also as $x \to \infty$ $\sinh x \to \infty$ and

$x \to -\infty$ $\sinh x \to -\infty$

$$d/dx(\sinh x) = \cosh x$$

$\cosh x$ always > 0

$\therefore$ curve rises

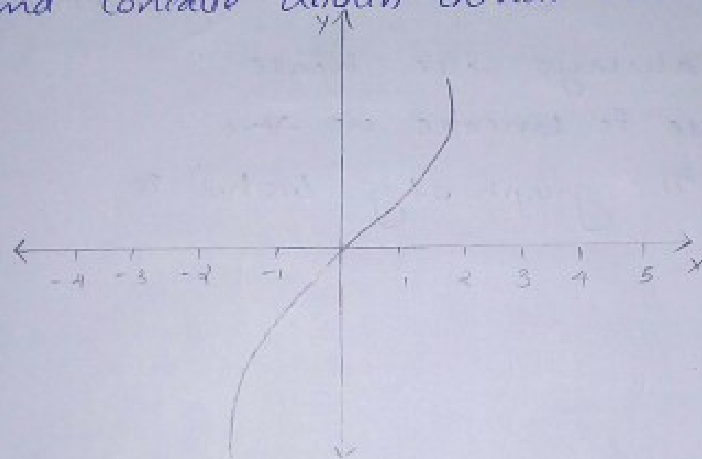
Also Second derivative

$$\frac{d^2}{dx^2}(\sinh x) = \sinh x$$

$\sinh x < 0 \ \forall x < 0$ and

$\sinh x > 0 \ \forall x > 0$

Then the curve is Concave up when $x>0$ and Concave down when $x<0$.



→ Graph of $y = \cosh x$.

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$\cosh x$ is always +ve

also $\cosh(-x) = \frac{e^{-x} + e^{-(-x)}}{2}$

$$= \frac{e^x + e^{-x}}{2} = \cosh x$$

ie $\cosh(-x) = \cosh x$

hence hyperbolic cosine is an even function and Symmetric about y-axis

$$\cosh 0 = \frac{e^0 + e^{-0}}{2} = \frac{1+1}{2} = \underline{\underline{1}}$$

hence $y = \cosh x$ passes through $(0,1)$ also

$x \to \infty$ or $x \to -\infty$ $\cosh x \to \infty$

the Second derivative of $y = \cosh x$

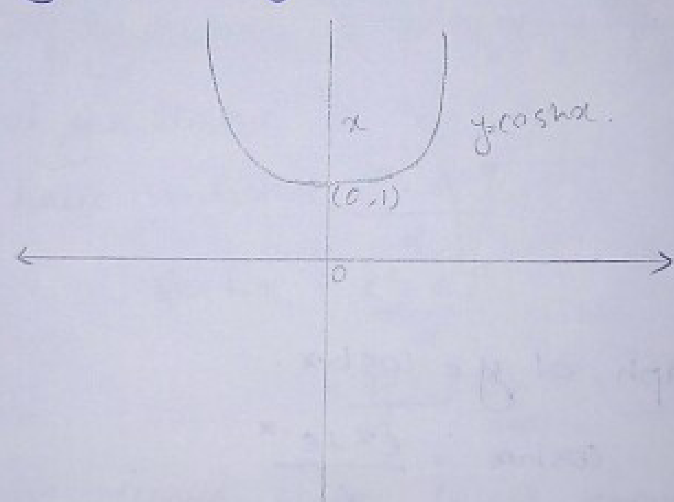
$$y' = \sinh x$$

$$y'' = \cosh x$$

is always +ve hence

curve is concave up $\forall x$

$\therefore$ The graph of $y = \cosh x$ is



→ Expression for inverse hyperbolic function in terms of logarithms.

(1)

$$y = \sinh^{-1} x$$

$$x = \sinh y = \frac{e^y - e^{-y}}{2}$$

$$2x = e^y - e^{-y}$$

Multipling throughout by e^y

$$2x e^y = e^{2y} - 1$$

$$e^{2y} - 2xe^y - 1 = 0.$$

which is a quandratic equation in e^y

hence $e^y = \dfrac{2x \pm \sqrt{4x^2 - 4 \times 1 \times -1}}{2 \times 1}$

$$= \frac{2x \pm \sqrt{4x^2+4}}{2}$$

$$= x \pm \sqrt{x^2+1}$$

ie, $e^y = x + \sqrt{x^2+1} \Rightarrow y = \ln(x + \sqrt{x^2+1})$

hence $\sinh^{-1}x = y = \ln(x+\sqrt{x^2+1})$ $\forall$ real x.

(2) $\cosh^{-1}x$ in terms of logarithm

$$y = \cosh^{-1}x \quad \text{where } x \geq 1$$

Then $x = \cosh y$

$$= \frac{e^y + e^{-y}}{2}$$

$$2x = e^y + e^{-y}$$

Multiplying both sides by e^y and rearr-anging terms we have

$$2xe^y = e^{2y} + 1$$

$$e^{2y} - 2xe^y + 1 = 0$$

$$e^y = \frac{2x \pm \sqrt{4x^2 - 4 \times 1 \times 1}}{2}$$

$$= \frac{2x \pm 2\sqrt{x^2 - 1}}{2}$$

$$= x \pm \sqrt{x^2 - 1}$$

$$y = \ln(x + \sqrt{x^2 - 1})$$

$$y = \cosh^{-1}x = \ln(x + \sqrt{x^2 - 1})$$

3) ~~tanh⁻¹x~~ $\tanh^{-1}x$ in terms of logarithms.

Let $y = \tanh^{-1}x$ where $-1 < x < 1$

Then $x = \tanh y = \frac{e^y - e^{-y}}{e^y + e^{-y}}$

$$(e^y + e^{-y})x = e^y - e^{-y}$$

$$(e^{2y} + 1)x = e^{2y} - 1$$

$$e^{2y}x + x = e^{2y} - 1$$

$$(1 - x)e^{2y} = x + 1$$

$$e^{2y} = \frac{1 + x}{1 - x}$$

$$2y = \ln \frac{1 + x}{1 - x}$$

$$y = \frac{1}{2}\left(\ln \frac{1 + x}{1 - x}\right)$$

$\tanh^{-1}x = \frac{1}{2} \ln \frac{1 + x}{1 - x}$ for $-1 < x$

Similarly

$$\coth^{-1} x = \tfrac{1}{2} \ln \frac{x+1}{x-1} \quad |x|>1$$

$$\operatorname{sech}^{-1} x = \ln \frac{1+\sqrt{1-x^2}}{x} \quad 0 \le x \le 1$$

$$\operatorname{cosech}^{-1} x = \ln \frac{1+\sqrt{x^2+1}}{x} \text{ for } x>0$$

$$\ln \frac{1-\sqrt{x^2+1}}{x} \text{ for } x<0$$

Q. Prove that the inverse hyperbolic function satisfy the following identities.

(i) $\operatorname{sech}^{-1} x = \cosh^{-1}(\frac{1}{x})$

(ii) $\operatorname{cosech}^{-1} x = \sinh^{-1}(1/x)$

(iii) $\coth^{-1} x = \tanh^{-1}(1/x)$

→ (i) Let $y = \operatorname{sech}^{-1} x$

Then $x = \operatorname{sech} y$

$1/x = \frac{1}{\operatorname{sech} y}$ $\quad$ $1/x = \cosh y$

hence $y = \cosh^{-1}(1/x)$

ie, $\operatorname{sech}^{-1} x = \cosh^{-1}(1/x)$

(ii) Let $y = \operatorname{cosech}^{-1} x$

$\Rightarrow \operatorname{cosech} y = x$

$\frac{1}{\operatorname{cosech} y} = 1/x$

$\sinh y = 1/x$

$y = \sinh^{-1}(1/x)$

ie, $\cosh^{-1} x = \sinh^{-1}(1/x)$

(iii) Let $y = \coth^{-1} x$.

$\tanh y = x$

$\frac{1}{\tanh y} = 1/x$

$y = \tanh^{-1}(1/x)$

ie, $\coth^{-1} x = \tanh^{-1}(1/x)$

→ Derivative of Inverse hyperbolic functions.

$d/dx (\sinh^{-1} x)$

Let $y = \sinh^{-1} x$

$x = \sinh y$

Differentiate w.r.t to x

$1 = \cosh y \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{\cosh y}$

$\cosh^2 y = \sinh^2 y = 1$

$$\cosh y = \sqrt{1+\sinh^2 y}$$

$$= \sqrt{1+x^2} \quad (\because \sinh y = x)$$

$$\boxed{\frac{dy}{dx} = \frac{d}{dx}(\sinh^{-1}x) = \frac{1}{\sqrt{1+x^2}}}$$

(a) $\frac{d}{dx}(\cosh^{-1}x)$

Let $y = \cosh^{-1}x$

$x = \cosh y$

Differentiate both sides w.r.t. to

$$1 = \frac{1}{\sinh y}$$

Also $\cosh^2 y = \sinh^2 y = 1$

$\sinh^2 y = \cosh^2 y - 1$

$\Rightarrow \sinh y = \sqrt{\cosh^2 y - 1}$

$= \sqrt{x^2-1}$

$$\boxed{\frac{d}{dx}\cosh^{-1}x = \frac{1}{\sinh y} = \frac{1}{\sqrt{x^2-1}}}$$

Similarly

$\frac{d}{dx}(\tanh^{-1}x) = \frac{1}{1-x^2} \quad |x|<1$

$\frac{d}{dx}(\coth^{-1}x) = \frac{1}{1-x^2} \quad |x|>1$

$\frac{d}{dx}(\operatorname{sech}^{-1}x) = \frac{1}{x\sqrt{1-x^2}} \quad 0<x<1$

$$\frac{d}{dx}(\operatorname{cosech}^{-1}x) = -\frac{1}{|x|\sqrt{1+x^2}}, \quad x \neq 0.$$

Problems.

Differentiable the following function w.r.t x.

(1) $\sinh^{-1}(3x+\cos x)$

$$\frac{d}{dx}(\sinh^{-1}x) = \frac{1}{\sqrt{1+x^2}}$$

$$\frac{d}{dx}(\sinh^{-1}(3x+\cos x)) = \frac{1}{\sqrt{1+(3x+\cos x)^2}} \times \frac{d}{dx}(3x+\cos x)$$

$$= \frac{1}{\sqrt{1+(3x+\cos x)^2}} \times 3-\sin x$$

$$= \frac{3-\sin x}{\sqrt{1+(3x+\cos x)^2}}$$

(2) $\frac{d}{dx}(\cosh^{-1}\sqrt{x^2+4})$

$\cosh^{-1}(\sqrt{x^2+4})$

$$\frac{d}{dx}(\cosh^{-1}x) = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cosh^{-1}\sqrt{x^2+4}) = \frac{1}{\sqrt{(x^2+4)^2-1}} \times \frac{d}{dx}\sqrt{x^2+4}$$

$$= \frac{1}{\sqrt{(x^2+4)^2-1}} \times \frac{1}{2\sqrt{x^2+4}}$$

(2) $\frac{d}{dx}\left(\cosh^{-1}\sqrt{x^2+1}\right)$

$$\frac{d}{dx}\cosh^{-1}x = \frac{1}{\sqrt{x^2-1}}$$

$$= \frac{1}{\sqrt{x^2+1-1}} \times \frac{d}{dx}\left(\sqrt{x^2+1}\right)$$

$$= \frac{1}{\sqrt{x^2}} \times \frac{1}{2\sqrt{x^2+1}} \times \frac{d}{dx}x^2+1$$

$$= \frac{1}{x} \times \frac{1}{\cancel{2}\sqrt{x^2+1}} \times \cancel{2}x$$

$$= \frac{1}{\cancel{x}} \times \frac{\cancel{x}}{\sqrt{x^2+1}}$$

$$= \frac{1}{\sqrt{x^2+1}}$$

(3) $\frac{d}{dx}\left(x^2\coth^{-1}(x+1)\right)$

$$= x^2 \times \frac{1}{1-(x+1)^2} + 2x \times \coth^{-1}(x+1)$$

$$= x^2 \times \frac{1}{1-x^2+2x+1} + 2x \times \coth^{-1}(x+1)$$

$$= \frac{x^2}{x^2+2x} + 2x \times \coth^{-1}(x+1)$$

$$\frac{x^2}{x(x+2x)} + 2x \times (\coth^{-1}(x+1)$$

$$\frac{x}{x+2}$$

(3) $\frac{d}{dx}\, x^2 \cot^{-1}(x+1)$

$$= \frac{d}{dx}(x^2) \cdot \cot^{-1}(x+1) + x^2 \cdot \frac{d}{dx}(\cot^{-1}(x+1))$$

$$= 2x \cot^{-1}(x+1) + x^2 \cdot \frac{1}{1-(x+1)^2} \cdot \frac{d}{dx}(x+1)$$

$$= 2x \cot^{-1}(x+1) + \frac{x^2}{-x^2-2x} \cdot 1$$

(4) $\int \tanh 5x \, dx$

$$= \int \frac{\sinh x}{\cosh x}$$

put $u = \cosh 5x$

$du = \sinh 5x + 5dx$

$$\int \tanh 5x = \int \frac{1}{5u} \cdot du$$

$$= \frac{1}{5} \ln u + c$$

$$= \frac{1}{5} \ln|\cosh 5x| + c$$

(2) $\int_0^{\ln 2} 4e^x \sinh x \, dx$

$$= 4\, e^x \left(\frac{e^x - e^{-x}}{2}\right)$$

$$= \int 2(e^{2x} - 1)$$

$=$

(3) $\cancel{d/dx(\cosh^{-1}x)}$ $\int \frac{dx}{\sqrt{4x^2-1}}$

$\rightarrow$

$$= \int \frac{dx}{(2x)^2-1}$$

$$u = 2x$$

$$\frac{du}{dx} = 2$$

$$du = 2dx$$

$$\int \frac{dx}{\sqrt{4x^2-1}} = \int \frac{du}{2} \times \frac{1}{\sqrt{u^2-1}}$$

$$= \frac{1}{2} \cosh^{-1} u + c$$

$$= \frac{1}{2} \cosh^{-1} 2x + c$$

$\rightarrow$ Indeterminate forms.

In limits and continuity it was shown that if $L = \lim_{x \to c} f(x)$ and $M = \lim_{x \to c} g(x)$ and if $M \neq 0$ then $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} = L/M$

However if $M=0$ and $L\neq 0$ we usually take the limit to be infinite

In the case $L=0$, $M=0$ the limit of the quotient f/g is said to be indeterminate

<u>Theorem</u>:- L Hopitals rule- Stronger form.

Suppose that $f(a)=g(a)=0$ and that f and g are differentiable on an open interval I containing a. Suppose also that $g'(x)\neq 0$ if $x\neq 0$ then $\lim_{x\to 0} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$ if the limit exist on the right exist.

(1) Evaluate $\lim_{x\to 1} \frac{1-x}{\ln x}$

$\rightarrow$ $\lim_{x\to 1} \frac{1-x}{\ln x}$ ($0/0$ for apply l hopitalis rule)

$$= \lim_{x\to 0} \frac{0-1}{1/x}$$

$$= \lim_{x\to 1} -x = -1$$

(2) $\lim_{x\to 0} \frac{x-\sin x}{x^3}$

$$= \lim_{x\to 0} \frac{1-\cos x}{3x^2}$$

$$= \lim_{x\to 0} \frac{0-\sin x}{6x}$$

$$= \lim_{x \to 0} \frac{\sin x}{6x}$$

$$= \lim_{x \to 0} \frac{\cos x}{6}$$

$$= 1/6$$

3. $\lim_{x \to 0} \frac{1 - \cos x}{x + x^2}$ (0/0 form)

$$= \frac{1 + \sin x}{1 + 2x} = \frac{0}{1 + 2 \times 0} = 0/1$$

$$= \frac{\cancel{1 + \sin 0}}{\cancel{1 + 2 \times 0}} = \cancel{1/1} = 0$$

4. $\lim_{x \to 0} \frac{\sin x}{x^2}$ (0/0 form)

$$= \frac{\cos x}{2x} \quad (1/0)$$

$$= \frac{\cancel{-\sin x}}{\cancel{2}} = \infty$$

$$= \cancel{-\cos x}$$

5. $\lim_{x \to 0} \frac{x^2}{1 - \cos x} = \lim_{x \to 0} \frac{2x}{0 + \sin x}$ (0/0)

$$= \frac{2}{\cos x}$$

$$= \frac{2}{1} = 2$$

$0/0$ form Q $\lim_{x \to 0} \frac{\ln x}{\cot x}$

$\rightarrow \lim_{x \to 0} \frac{\ln x}{\cot x} = \lim_{x \to 0} \frac{1/x}{-\operatorname{cosec}^2 x}$

$= \lim_{x \to 0} 1/x \times -\sin^2 x$

$= \lim_{x \to 0} \frac{\sin^2 x}{x}$

$= \lim_{x \to 0} \frac{2 \sin x \cos x}{1}$

$= 0$ //

$\infty - \infty$ form.

Q. $\lim_{x \to 0} \left(\frac{1}{\sin x} - 1/x \right)$

$\rightarrow \lim_{x \to 0} \frac{1}{\sin x} - 1/x = \lim_{x \to 0} \frac{x - \sin x}{x \sin x}$

$= \lim_{x \to 0} \frac{1 - \cos x}{x \times \cos x + \sin x \times 1}$

$= \lim_{x \to 0} \frac{\sin x}{\cos x + \cos x + x \times -\sin x}$

$= \lim_{x \to 0} \frac{\sin x}{2 \cos x - x \sin x}$

$= \frac{0}{2 - 0} = 0$

Q Evaluate the following limits

(i) $\lim_{x \to 2} \frac{x-2}{x^2-4}$

(ii) $\lim_{x \to 0} \frac{8x^2}{\cos x - 1}$

(iii) $\lim_{x \to \pi/2} x \tan x - \frac{1}{2}\pi \sec x$.

→ (i) $\lim_{x \to 2} \frac{x-2}{x^2-4}$ ($0/0$ form)

$\lim_{x \to 2} \frac{x-2}{x^2-4} = \lim_{x \to 2} \frac{1-0}{2x-0} = \frac{1}{2x}$

$= \frac{1}{4}$ •

$\lim_{x \to \infty} \frac{\ln x}{2\sqrt{x}}$

(ii) $\lim_{x \to 0} \frac{8x^2}{\cos x - 1}$ $(0/0)$ $= \lim_{x \to 0} \frac{16x}{-\sin x - 0}$ ($0/0$ form)

$= \lim_{x \to 0} \frac{16}{-\cos x}$

$= \frac{16}{-1}$ •

(iii) $\lim_{x \to \pi/2} x \tan x - \frac{1}{2}\pi \sec x$