CHAPTER –II

WAVE PROPERTIES OF PARTICLES

Introduction

In 1924, Louis de Broglie put forward the suggestion that matter, like radiation has dual nature ie. matter which is made up of discreet particles, atoms, protons, electrons etc. might exhibit wave like properties under appropriate conditions. These waves associated with a material particle are called "matter waves".

De Broglie hypothesis of Matter Waves.

We know that light posses dual nature, behaving as a wave as well as a particle (Photon). In analogy with dual character of light, de Broglie in 1924 argued that since nature loves symmetry and simplicity in physical phenomena all material particles should exhibit both wave ad particle nature.

According to de Broglie hypothesis, a moving particle, whatever its nature, has wave properties associated with it. i.e. particles such as electrons, protons etc have waves associated with them. These waves are called matter waves or de Broglie waves.

De Broglie Waves

A moving body behaves in certain ways as though it has a wave nature. Consider a photon of light of frequency ϑ.

Its momentum $P = \frac{h\vartheta}{c}$

But $c = \vartheta\,\lambda$

$\therefore \quad P = \frac{h\vartheta}{\vartheta\,\lambda}$

or, $p = \frac{h}{\lambda}$

$$E = h\vartheta$$

$$E = mC^2$$

$$h\vartheta = mc^2 = mc.c$$

$$h\vartheta = p.c \quad [\therefore P = Mass \times Vel = m \times c]$$

$$or,\; P = \frac{h\vartheta}{c}$$

∴ the wavelength associated with a photon,

or photon wave length $\lambda = \frac{h}{p} \ldots \ldots \ldots \ldots (1)$ ***-- This is the de- Broglie relation***

This eqn can be applied to material particles as well as to photons.

Consider a particle of mass m moving with a velocity 'v'.

Its momentum P = m v

Hence its de Broglie wave length $\boxed{\lambda = \frac{h}{P} = \frac{h}{mv}}$ ………………….(2)

This is known as de. Broglie wave eqn and λ is called de- Broglie wave length. The associated wave is termed matter, guide, pilot, or de-Broglie waves. Here 'm' is the relativistic mass,

ie $m = \frac{m_0}{\sqrt{1 - \frac{V^2}{c^2}}}$

Summary

The waves associated with a material particle are called matter waves or de-Broglie waves. The wave length of the waves associated with material particles is called de Broglie wave length and is given by,

$$\lambda = \frac{h}{P} = \frac{h}{mv}$$

Note:

The wave and particle aspects of moving bodies can never be observed at the same time. ie. in certain situations a moving body exhibits wave properties and in others it exhibits particle properties.

Nature of de-Broglie Waves, Waves of Probability

1. De Broglie waves are probability Waves
2. The quantity whose variation makes up matter waves is called the wave function Ψ (psi). The value of the wave function associated with a moving body at the particulate point x, y, z in space at the time't' is related to the likelihood of finding the body there at the same time.

The wave function Ψ has no direct physical significance ie Ψ itself cannot be an observable quantity.

3. The square of the absolute value of the wave function is known as probability density $|\Psi|^2$. The probability of experimentally finding the body described by the wave function 'Ψ' at the point x, y, z at the time 't' is proportional to the value of $|\Psi|^2$.

Wave velocity or Phase Velocity 'V_P**'**

When a monochromatic wave (ie wave of single frequency and wavelength) travels through a medium its velocity of advancement in the medium is called the wave velocity.

eg,

A plane harmonic wave travelling along the +ve X direction is given by

$$y = a\, sin(\omega t - kx)$$

By definition,

a = amplitude

$\omega = 2\pi\vartheta$ = Angular frequency

$k = \frac{2\pi}{\lambda}$ = propagation constant of the waves.

Wave velocity $V_p = \frac{\omega}{k} = \frac{angular\ \ frequency}{propagation\ \ constant}$

$(\omega t - kx)$ = Phase of the wave motion.

Planes of constant phase (wave front) are defined by,

$\omega t - k\,x$ = constant

diff. with respect to 't'

$$\omega - k\frac{dx}{dt} = 0$$

$$\text{Or} \quad \frac{dx}{dt} = \frac{\omega}{k} = V_p$$

∴ the wave velocity is the velocity with which planes of constant phase advance through the medium. Hence the wave velocity is also called as phase velocity.

Group Velocity (V_g) or Velocity of group of waves

Group velocity is the velocity with which the slowly varying envelope of the modulated pattern due to a group of waves travels in a medium. Or

It is the velocity with which wave group is transmitted.

$$V_g = \frac{\Delta\omega}{\Delta k}$$

The Wave Equation

The de Broglie wave velocity or

Phase velocity $V_p = \vartheta\lambda$ ________(1)

Where $\lambda = \frac{h}{mv}$ __________(2)

But $h\vartheta = mc^2$

$$\vartheta = \frac{mc^2}{h}$$ ________(3)

$v = \vartheta\lambda$

λ = de Broglie wave length

v = vel. of the body or moving particle

$E = h\vartheta$ also, $E = mc^2$

put (2) and (3) in (1)

$$V_p = \frac{mc^2}{h} \frac{h}{mv} = \frac{c^2}{v}$$

Velocity of the de Broglie waves or de Broglie phase velocity

$$V_p = \frac{c^2}{v}$$

v<<c

Since the particle velocity 'v' is less than the velocity of light 'c', the de Broglie waves always travel faster than light (To understand this unexpected result, we must see the distinction between phase velocity and group velocity).

The wave equation for Specific place ie at $x = 0$ *is*

$$y = A\ \cos 2\pi\,\vartheta t$$

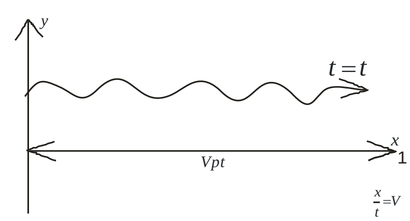
$y = A\,cos(\omega t - kx)$

$\vartheta = freq$

$\omega = 2\pi\,\vartheta$

$A = amplitude$

$y = displacement$



The wave travels a distance $x = V_p t$ in a time 't' ∴ the time interval between the formation of the wave at $x = 0$, and its arrival at the point x is $\frac{x}{V_p}$.

Hence the displacement y of the string at x at any time't' is exactly the same as the value of y at $x = 0$ at the earlier time $t - \frac{x}{V_p}$

∴ wave equation for any time and place

$$y = A\cos 2\pi\,\vartheta\left(t - \frac{x}{V_p}\right)$$

$$= A\cos 2\pi\left(\vartheta t - \frac{\vartheta x}{V_p}\right) \qquad V_p = wave\ speed$$

$$= A\cos 2\pi\left(\vartheta t - \frac{x}{\lambda}\right) \qquad V_p = \vartheta\lambda$$

∴ the wave eqn is

$$\boxed{y = A\cos 2\pi\left(\vartheta t - \frac{x}{\lambda}\right)}$$

But angular frequency $\omega = 2\pi\,\vartheta$

Wave number $k = \frac{2\pi}{\lambda}$

$$k = \frac{\omega}{V_p}$$

$$k = \frac{2\pi}{\lambda} \times \frac{\vartheta}{\vartheta} = \frac{\omega}{V_p}$$

∴ wave formula is $\boxed{y = A\cos\ (\omega t - kx)}$

Phase and Group Velocity

Wave group

A wave packet is the combination of many indivudual waves

A wave packet or wave group is the result of superposing waves of different wave lengths. In the case of de-Broglie waves, the wave velocity varies with wave length. Hence the different individual waves do not proceed together. ∴ the wave group has a velocity different from those of the waves that compose it.

Consider two waves that have the same amplitude “A” but differ by an amount Δw in angular frequency and an amount Δk in wave number. They can be represented by the eqns.

$$y_1 = Acos(\omega t - kx)$$

A=amplitude

ω = angular frequency

K= wave number

$$y_2 = Acos[(\omega + \Delta\omega)t - (k + \Delta k)x]$$

$\omega + \Delta\omega = angular\ freq$

$k + \Delta k = wave\ number$

A= same amplitude

The superposition of two waves will yield a single wave packet or wave group.

∴ resultant displacement

$$y = y_1 + y_2$$

$$= A\,cos(\omega t - kx) + Acos[(\omega + \Delta\omega)t - (k - \Delta k)x]$$

$cos\,A + cos\,B$

$= 2\,cos\frac{A+B}{2}\;cos\;\frac{A-B}{2}$

$$y = 2A\,cos\frac{1}{2}[(2\omega + \Delta\omega)t - (2k + \Delta k)x].cos\frac{1}{2}[(\Delta\omega t - \Delta kx)]$$

Since $\Delta\omega$ and Δk are small

$$2\omega + \Delta\omega \approx 2\omega$$

$$2k + \Delta k \approx 2k$$

also, $cos(-\theta) = cos\,\theta$

$$\therefore y = 2A\,cos(\omega t - kx)\,cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right) \text{……………………….(1)}$$

This is the analytic expression for resultant wave (wave packet) due to superposition of the two waves.

We know beats are produced by the superposition of two waves with different frequencies.

∴ Beat, y = $2A\cos(\omega t - kx)\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right)$

Eqn (1) represents a wave of angular frequency 'ω' and wave number 'k' that has superimposed upon it a modulation of angular frequency $\frac{1}{2}\Delta\omega$ and wave number $\frac{1}{2}\Delta k$

We know,

1. *Phase velocity* $V_p = \frac{\omega}{K}$ … … … … … … … … (2)
2. *Velocity of the wave group or group velocity* $V_g = \frac{\Delta\omega}{\Delta K}$ … … … … … … (3)

   When w and k have continuous spreads, group velocity $V_g = \frac{d\omega}{dk}$ … … … … … (4)

   This is the expression for group velocity.

Group velocity of de- Broglie waves: or a relation between group velocity and particle velocity.

According to de Broglie hypothesis a particle moving with velocity 'v' is supposed to consist of a group of waves,

Then group velocity $V_g = \frac{d\omega}{dk}$

Angular frequency of de Broglie waves,

$$\omega = 2\pi\vartheta = \frac{2\pi mc^2}{h} = \frac{2\pi m_0 c^2}{h\sqrt{1-v^2/c^2}} \text{……………………(1)}$$

$$E = h\vartheta$$

$$E = mc^2$$

$$\therefore h\vartheta = mc^2 \therefore v = \frac{mc^2}{h}$$

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Wave number of the Broglie waves

$$k = \frac{2\pi}{\lambda} = \frac{2\pi mv}{h} = \frac{2\pi m_0 v}{h\sqrt{1-v^2/c^2}} \text{………………………(2)}$$

$$\lambda = \frac{h}{mv}$$

Group velocity V_g of the de- Broglie waves associated with the particle is

$$V_g = \frac{d\omega}{dk} = \frac{\frac{d\omega}{dv}}{dk/_{dv}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (3)$$

$$\frac{d\omega}{dv} = \frac{d}{dv}\left[\frac{2\pi m_0 c^2}{\mathrm{h}\sqrt{1-v^2/_{c^2}}}\right] = \frac{d}{dv}\left[\frac{2\pi m_0 c^2}{h}\left(1 - v^2/_{c^2}\right)^{-1/_2}\right]$$

$$= \frac{2\pi m_0 c^2}{h}\left[\frac{-1}{2}\left(1 - v^2/_{c^2}\right)^{-3/2} \times \frac{-2v}{c^2}\right]$$

$$\frac{d\omega}{dv} = \frac{2\pi m_0 v}{h}\frac{1}{\left(1 - v^2/_{c^2}\right)^{3/_2}} \dots\dots\dots\dots\dots\dots\dots\dots (3)\ a$$

also, $\frac{dk}{dv} = \frac{d}{dv}\left[\frac{2\pi m_0 v}{\mathrm{h}\sqrt{1 - v^2/_{c^2}}}\right] = \frac{2\pi m_0 v}{h}\left[1 - v^2/_{c^2}\right]^{-1/_2}$

$$= \frac{2\pi m_0}{\mathrm{h}}\left[1 \times \left(1 - v^2/_{c^2}\right)^{-1/_2} + v \times -1/_2\left(1 - v^2/_{c^2}\right)^{-3/_2} \times \frac{-2v}{c^2}\right]$$

$$= \frac{2\pi m_0}{\mathrm{h}}\left[\left(1 - v^2/_{c^2}\right)^{-1/_2} + \frac{v^2}{c^2}\left(1 - v^2/_{c^2}\right)^{-3/_2}\right]$$

$$= \frac{2\pi m_0}{\mathrm{h}}\left(1 - v^2/_{c^2}\right)^{-3/_2}\left[\left(1 - v^2/_{c^2}\right) + \frac{v^2}{c^2}\right]$$

$$\frac{dk}{dv} = \frac{2\pi m_0}{\mathrm{h}}\frac{1}{\left(1-v^2/_{c^2}\right)^{3/_2}} \dots\dots\dots\dots\dots\dots\dots\dots (3)\ b$$

Put (3) a and (3) b in (3), we get,

$$\therefore V_g = \frac{d\omega/_{dv}}{dk/_{dv}} = v$$

∴ de Broglie group velocity V_g = v………………………(4)

ie. The de Broglie wave group associated with a moving body travels with the same velocity as the particle. ***Thus a moving particle is equivalent to a wave group***.

Relation between group velocity V_g and Wave Velocity or phase velocity V_p

We have,

Wave Velocity $V_p = \frac{\omega}{k}$.....................................(5)

Group velocity $V_g = \frac{d\omega}{dk}$...........................(6) | or $\omega = kV_p$

$$\therefore V_g = \frac{d}{dk}\left(kV_p\right) = V_p + k\frac{dV_p}{dk} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

but, $k = \frac{2\pi}{\lambda}$, $\therefore dk = \frac{-2\pi}{\lambda^2}d\lambda$

$$\therefore \frac{k}{dk} = \frac{2\pi/\lambda}{-2\pi/\lambda^2 \, d\lambda} = \frac{-\lambda}{d\lambda}$$

put this in eqn (7)

$$V_g = V_p + k\frac{dV_p}{dk} = = V_p + \left(\frac{-\lambda}{d\lambda}\, dV_p\right)$$

Or

$$\boxed{V_g = V_p - \lambda\frac{dV_p}{d\lambda}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

This is the relation between group velocity V_g and phase or wave velocity V_p

Note

1. De Broglie phase velocity $V_p = \frac{\omega}{k}$

   Put eqn (1) and (2) in the above equation then we get,

   $V_p = \frac{c^2}{V}$

   This exceeds both the velocity of the body ‘v’ and the velocity of light ‘c’. The wave velocity V_p of de- Broglie waves has actually no Physical significance at all. | v < c

2. ***Conclusion***

The wave or phase velocity corresponds to the velocity of the individual waves comprising the group where as the group velocity stands for the velocity at which the energy actually travels.

Particle Diffraction

An essential property of a wave is that it can undergo diffraction. According to classical physics, the property of diffraction is not shown by particles. But the existence of de- Broglie waves associated with moving particles is verified by experiments on the diffraction of electrons. These experiments are based on the fact that if material particles have a wave character, they should show the phenomenon of diffraction. Experiments show that electrons do exhibit diffraction effects, when they (electron) are scattered from crystals. This is known are particle diffraction.

1. ***Experimental study of matter waves.***

Direct evidence of the existence of de Broglie waves are furnished by experiments on the diffraction of electrons.

Consider an electron accelerated through a potential '**V**' volt, and then its K.E acquired,

$$= \frac{1}{2} m_0 v^2 = e\boldsymbol{V}$$

Or $$v = \sqrt{\frac{2e\mathbf{V}}{m_0}}$$

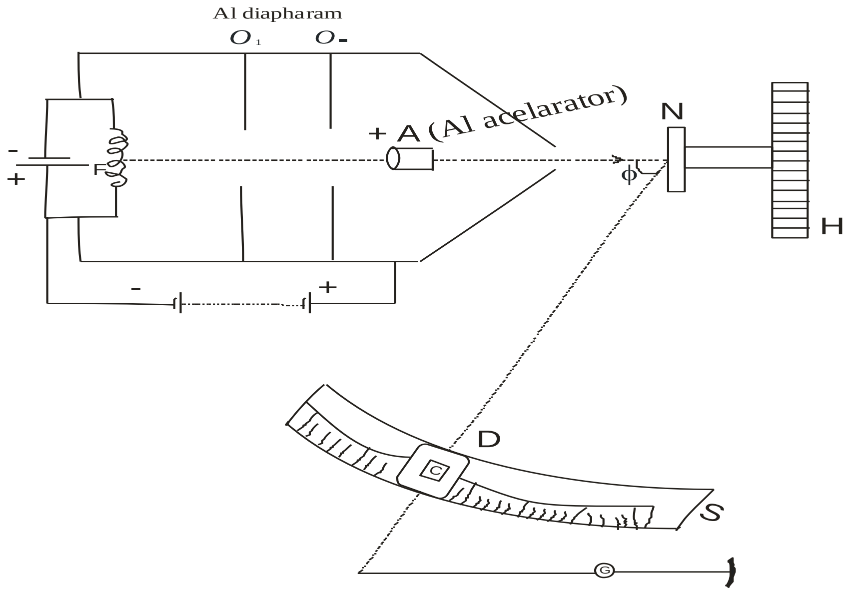
but $$\lambda = \frac{h}{m_0 v} = \frac{h}{m_0 \sqrt{\frac{2e\mathbf{V}}{m_0}}}$$

$$\lambda = \frac{h}{\sqrt{2 m_0 e\mathbf{V}}} = \frac{12.27}{\sqrt{\mathbf{V}}} A^0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Since this wavelength is comparable with the wave length of X – rays, the de-Broglie waves should be capable of being diffracted by a crystal, just as X - rays are. Eqn (1) provides the theoretical formula of Davisson and Germer's experiment.

Davisson and Germer's Experiment

Davisson and Germer's experiment provides a direct verification of de-Broglie hypothesis of the wave nature of moving particles.

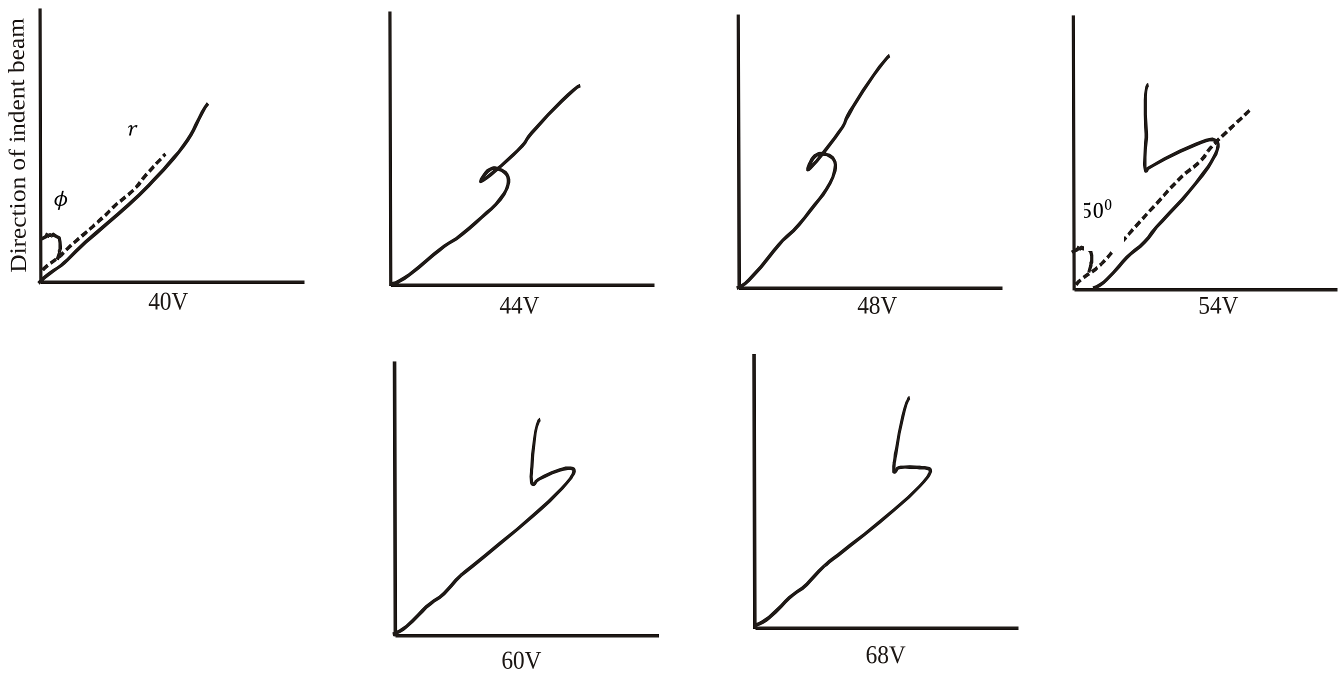


Experimental arrangement

Experiment arrangement is shown in figure; electrons are produced by heating a filament ‘F’ by a L.T battery. Electrons are collimated to fine beam by thin aluminum diaphragms D_1 and D_2. The electrons are then accelerated by passing them through an aluminum cylinder ‘A’, to which the required high potential (H.T) can be applied. This electron beam falls on a large single crystal of nickel N. The crystal is capable of rotation about an axis parallel to the axis of the incident beam by a handle H. The electrons are scattered in all directions by the atoms in the crystal. The electrons scattered in different directions are collected by a Faraday cylinder C, called the collector. The collector is connected to a sensitive galvanometer G and can be moved along a graduated circular scale ‘S’ so that it is able to receive reflected electrons at all angles between 20^0 and 90^0. The galvanometer deflection is proportional to the intensity of the beam entering the collector.

Experiment procedure and calculations

Let a beam of electrons be made to all fall normally on the crystal. The collector is moved to various positions on the scale ‘S’, and the galvanometer defection at each position is noted. The galvanometer deflection is plotted against colatitude (the angle between the incident beam and the beam entering the collector is called colatitude). The observations are repeated for different accelerating voltages and a number of curves are drawn.



The graph remains smooth till the p.d becomes 44V when a spur (a little bump) appears on the curve. As the p.d is increased, the length of the spur increases till it reaches a maximum at 54V at an angle of 50^0. With further increase in accelerating voltage the spur decreases in length and finally disappears at 68 V.

The most prominent bump at 54 volts provides an evidence that electrons are associated with waves which after scattering from the atomic planes of the crystal give rise to constructive interference.

The de Broglie wave length associated with a 54 volt electron is.

$$\lambda = \frac{h}{m_0 V} = \frac{12.27^0 A}{\sqrt{V}} = \frac{12.27^0 A}{\sqrt{54}} = 1.66^0 A \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

This is the theoretical value of wave length of electron from de Broglie's relation. X-ray analysis shows that nickel crystal acts as a plane diffraction grating with grating space d $=.91^0$A

Using Bragg's eqn

$2d \sin\theta = n\lambda$

$\lambda = 2 \times .91\ \sin 65$

$\theta = \frac{180-50}{2}$

$\therefore\ \theta = 65^0$

n=1 first order

or $\lambda = 1.65^0 A \dots\dots\dots\dots\dots (3)$

This is the value of λ obtained from diffraction experiment.

Conclusion

Comparing eqn (2) and (3) we can say that the experimental value of λ is in close agreement with its theoretical value calculated from de Broglie's relation. Thus Davisson - Germer experiment provides direct experimental proof of de Broglie's hypothesis for the wave nature of moving particles. ie electron beams do behave as waves and the wave length of these beams is that given by the de Broglie eqn.

Electron Microscope

The wave nature of moving electrons is the basis of the electron microscope. An electron microscope is a device for magnifying very minute objects.

Principle

A particle of mass 'm' moving with a large velocity behaves like a wave of wave length $\lambda = \frac{h}{mv}$. This wave nature of the particle is used in the construction of an electron microscope. An electron microscope is in principle similar to that of an optical microscope. In an optical microscope using visible light, focusing is done by suitable lenses where as is an electron microscope a beam of electron is used in place of ordinary light and focusing is done by electric and magnetic focusing.

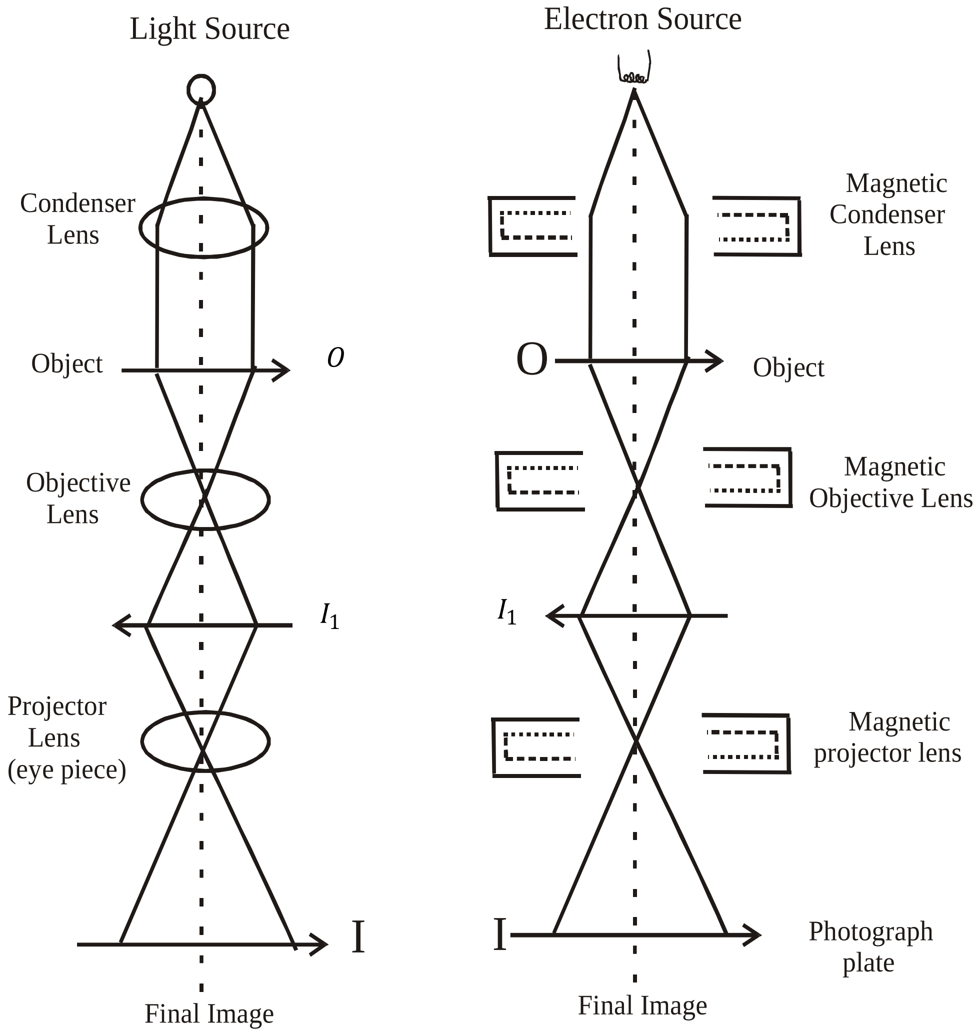
Construction

The construction of electron microscope is based on:

a) *The wave nature of moving electron*
b) *The ability of electric and magnetic field to cause deviation of an electron beam. This property is utilized in the construction of electric and magnetic lenses which focus and magnify high energy electron beams in the same way as a beam of light is focused and magnified by glass lenses.*

The working of an electron microscope is similar to that of an optical microscope. In the figure both are shown for comparison.

In an electron microscope a stream of electrons is obtained by an electron source. These electrons are accelerated by a p.d of about 60,000 volts. Electrostatic or magnetic lenses are used for focusing the beam on the object to be magnified.



The electrons scattered by the object are moving to the magnetic objective lens which forms a magnified image. Then the magnetic projector lens produces the final enlarged image. This image is received on a fluorescent screen where it is visible. The image may also be photographed.

For an electron of mass 'm' moving with a velocity 'v', the wave length of the associated wave is $\lambda = \frac{h}{mv}$.

For electron accelerated by a p.d of about 60000 volts, $\lambda = 5 \times 10^{-12} m$. This is 10^5 times smaller than the wave length of visible light (for yellow light, $\lambda \cong 6 \text{ x} 10^{-7} m$).

The limit of resolution (R.P) of a microscope is proportional to the wave length 'λ' . The resolving power of the electron microscope should be 10^5 times greater than that of the optical microscope.

Uses:

1. *In physics it has been used in the study of atomic structure and structure of crystals.*
2. *In medicine and biology it is used to study viruses and bacteria.*

Uncertainty Principle I

Statement

Heisenberg's uncertainty principle states that it is impossible to know both the exact position and momentum of an object at the same time.

ie the product of uncertainty Δx in the position of the body at some instant and the uncertainty Δp_x in its component of momentum in the x direction at the same instant is equal to or greater than $\frac{h}{4\pi}$

ie $\Delta x\, \Delta p \geq \frac{h}{4\pi}$

Δx= error in determining its postion

Δx=error in determining its momentum at the same instant

Proof:

Consider a wave group. The relationship between the distance Δx and the wave number spread Δk may be expressed as

$\Delta x\, \Delta k \geq {}^1/_2$ ………………..(1) by Fourier transform of the wave group

but $\lambda = \frac{h}{p}$

Wave number $k = \frac{2\pi}{\lambda} = \frac{2\pi P}{h}$

ie an uncertainty Δk in the wave number of the de Broglie waves associated with the particles results in an uncertainty ΔP in the particles momentum.

$\therefore \Delta k = \frac{2\pi}{h} \Delta p \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2)$

Put (2) in (1)

$$\Delta x \frac{2\pi}{h} \Delta p \geq \frac{1}{2}$$

$$\Delta x\, \Delta p \geq \frac{h}{4\pi}$$

∴ Uncertainty principle $\Delta x\, \Delta p \geq \frac{h}{4\pi}$

This equation states that the product of the uncertainty Δx in the position of an object at some instant and the uncertainty Δp in its momentum component in the x direction at the same instant is equal to or greater than $\frac{h}{2\pi}$. If we arrange matters so that Δx is small, corresponding to a narrow wave group, then Δp will be large. If we reduce Δp in some way, a wide wave group is inevitable and Δx will be large

Let $\hbar = \frac{h}{2\pi}$ | $\hbar = h - bar$

∴ Uncertainty principle,

$$\boxed{\Delta x\, \Delta p \geq \frac{\hbar}{2}}$$

Uncertainty Principle II--- by Particle approach

Consider the particle nature of waves, instead of the wave nature of particles and let us try to examine an electron with the help of light of wave length λ. In this process a photon of light strikes the electron and bounce off it. Each Photon posses the momentum $\frac{h}{\lambda}$ and when it collides with the electron, the electron's original momentum 'P' is changed. The precise change cannot be predicted. But it is likely to be of the same order of magnitude as the photon momentum $\frac{h}{\lambda}$. Hence the act of measurement introduce an uncertainty of $\Delta P \approx \frac{h}{\lambda}$...........(1) ,in the momentum of the electron. This eqn shows that, the larger the wave length of the light we employ to see the electron, the smaller the consequent uncertainty in its momentum.

Because light has wave properties, we cannot expect to determine the electrons position with infinite accuracy under any circumstances. But we can reasonably hope to keep the irreducible uncertainty Δx in its position to the wave length of the light used.

i.e $\Delta x \geq \lambda \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$

This eqn shows that the shorter the wavelength, the smaller the uncertainty in the position of the electron.

From eqns (1) and (2) it is clear that if we employ light of shorter wavelength to improve the accuracy of position determination, the accuracy of momentum determination will be reduced. Light of larger wavelength will yield an accurate momentum value but an inaccurate position value.

Combine eqns (1) and (2)

$$\Delta x\, \Delta P \geq\ \Delta P \lambda$$

$$\Delta x\ \ \Delta P \geq \frac{h}{\lambda}\lambda\ or\ \Delta x\, \Delta P \geq h \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Note

More usually we will consider

1. $\Delta x\, \Delta p \geq \hbar$
2. $\Delta x\, \Delta P \geq h$

$\hbar = \frac{h}{2\pi}$

Time Energy uncertainty relation

Time energy uncertainty principle states that "in any simultaneous determination of time and energy of a particle ,the product of the uncertainties is equal to or greater than Plank's const. h"

ie $\Delta E\ \Delta t\ \geq h$

Where ΔE= *uncertainity in the measurement of energy*

Δt= *the corresponding uncertainty in the measurement of time*

Proof:

Consider a electron of mass is moving with a velocity v.

Its $K.E = E = \frac{1}{2}\, mv^2 = \frac{1}{2m}(mv)^2 = \frac{P^2}{2m}$ | P = mv

or E= $\frac{p^2}{2m} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$

The uncertainty in the measurement of E is

$$\Delta E = \Delta\left(\frac{p^2}{2m}\right) = \frac{2p\Delta p}{2m} = \frac{p\Delta p}{m} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

but P = mv = m $\frac{\Delta x}{\Delta t}$ | v = $\frac{\Delta x}{\Delta t}$

put this in (2)

$$\Delta E = \frac{m\frac{\Delta x}{\Delta t}\Delta p}{m}$$

or , $\Delta E\, \Delta t = \Delta x\, \Delta P \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$

But we know $\Delta x\, \Delta p\ \ \geq h$

$$\therefore \Delta E\, \Delta t \quad \geq h \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

More accurately $\Delta E \Delta t \geq \frac{\hbar}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$

Applying the uncertainty principle

Planck's const h is so small that the limitations imposed by the uncertainty principle are significant only in the realm of the atom. On such a scale this principle is of great help in understanding many phenomena.

Example

Non- existence of electron in the nucleus

The radius of the nucleus is of the order of 10^{-14} m radius $\therefore$ if we assume that electron is confined within the nucleus, the uncertainty in its position will be $\Delta x = 2 \times 10^{-14} m$ (equal to the nuclear diameter)

We have,

$$\Delta x\, \Delta p = \hbar$$

$$\Delta p \quad = \frac{\hbar}{\Delta x} = \frac{1.054 \times 10^{-34}}{2 \times 10^{-14}} = 5.27 \times 10^{-21} kg - m/s$$

The minimum uncertainty in momentum can be taken as the momentum of the electron,

i.e $p = 5.27 \times 10^{-21} kg - {}^{m}/_{s}$

$\Delta p \approx P$
$m = 9.1 \times 10^{-31} kg$

$$\therefore E = \frac{p^2}{2m} = \frac{(5.27 \times 10^{-21})^2}{2 \times 9.1 \times 10^{-31}}$$

$$= \left[\frac{(5.27 \times 10^{-21})^2}{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19}}\right] ev$$

$$= 95.37 \times 10^{6} ev \; = \; 95.37 \text{ Mev}$$

It means that if electron exists inside the nucleus, its K.E is of the order of 95.37 MeV. But experimental results have shown that no electron in the atom possess energy greater than 4MeV. Hence we can say that electrons do not exist in the nucleus.

By k.c.a