» centripetal Acceleration

The position vector at any time t is given by

$r(t) = r_0 \cos\omega t \; i + r_0 \sin\omega t \; j$ where r_0 and ω are constants.

Then the velocity vector $v(t)$ is defined by

$$\boxed{v(t) = -r_0\omega \sin\omega t \; i + r_0\omega \cos\omega t \; j}$$

and acceleration vector $a(t)$ is defined by

$$\boxed{a(t) = -\omega^2 r(t)}$$

Let $v = \|v(t)\|$ and $a = \|a(t)\|$. Then

$$v = \|v(t)\| = \|-r_0\omega \sin\omega t\, i + r_0\omega\cos\omega t\, j\|$$

$$= \sqrt{(-r_0\omega\sin\omega t)^2 + (r_0\omega\cos\omega t)^2}$$

$$= \sqrt{r_0^2\omega^2\sin^2\omega t + r_0^2\omega^2\cos^2\omega t}$$

$$= \sqrt{r_0^2\omega^2[\sin^2\omega t + \cos^2\omega t]}$$

$$= \sqrt{r_0^2\omega^2 \times 1} \qquad [\because \sin^2\omega t + \cos^2\omega t]$$

$$= \sqrt{r_0^2\omega^2}$$

$$v = r_0\omega$$

$$a = \|a(t)\| = \|-r_0\omega^2\cos\omega t\, i - r_0\omega^2\sin\omega t\, j\|$$

$$= \sqrt{(-r_0\omega^2\cos\omega t)^2 + (-r_0\omega^2\sin\omega t)^2}$$

$$= \sqrt{r_0^2\omega^4\cos^2\omega t + r_0^2\omega^4\sin^2\omega t}$$

$$= \sqrt{r_0^2\omega^4[\cos^2\omega t + \sin^2\omega t]}$$

$$= \sqrt{r_0^2\omega^4 \times 1} \qquad [\because \cos^2\omega t + \sin^2\omega t = 1]$$

$$= \sqrt{r_0^2\omega^4}$$

$$a = r_0\omega^2$$

$$a = \frac{r_0}{r_0}[r_0\omega^2] = \frac{r_0(r_0\omega^2)}{r_0} = \frac{r_0^2\omega^2}{r_0}$$

$$a = \frac{r_0^2\omega^2}{r_0} = \frac{v^2}{r_0} \qquad [\because v = r_0\omega]$$

$$\therefore \boxed{a = \frac{v^2}{r_0}} \Rightarrow \text{centripetal Acceleration}$$

Motion of a Projectile

We know that $v'(t) = a(t)$. Hence integrating this equation with respect to t, we get

$$\int v'(t)\,dt = \int a(t)\,dt$$

$$v(t) = \int a(t)\,dt$$

Let $v(0) = v_0$ be the initial condition. Then

$$\boxed{v(t) = \int a(t)\,dt + v_0} \Rightarrow \text{Velocity Vector}$$

Since $v(t) = r'(t)$, we can obtain $r(t)$ by integrating both sides of the equation with respect to t. Thus we obtain

$$\int v(t)\,dt = \int r'(t)\,dt$$

$$\int r'(t)\,dt = \int v(t)\,dt$$

$$r(t) = \int v(t)\,dt.$$

Let $r(0) = r_0$ be the initial condition. Then

$$\boxed{\cancel{r(t) = \text{[illegible]}\, a(t) + v_0 t + r_0}}$$

$$\boxed{r(t) = \int v(t)\,dt + v_0 t + r_0} \Rightarrow \text{Position Vector}$$

Find the Velocity and Position Vector of an object with the given acceleration, initial Velocity and Position

$a(t) = -32k$, $v(0) = i + 2j$, $r(0) = 128k$?

$\Rightarrow V_0 = i + 2j$

$r_0 = 128k$

$v(t) = \int a(t)\,dt = \int -32k\,dt = -32k \int dt = -32t\,k$

$\therefore v(t) = \int a(t)\,dt + V_0$

$v(t) = -32t\,k + i + 2j$

$v(t) = i + 2j - 32t\,k$ $\Rightarrow$ Velocity vector

$r(t) = \int v(t)\,dt = \int -32t\,k\,dt = -32k \int t\,dt$

$r(t) = -32k \times \frac{t^2}{2}$

$r(t) = -16t^2 k$

$\because r_0 = 128k$, $r(t) = \int v(t)\,dt + V_0 t + r_0$

$r(t) = -16t^2 k + (i + 2j)t + 128k$

$r(t) = ti + 2tj - 16t^2 k + 128k = ti + 2tj + (-16t^2 + 128)k$ $\Rightarrow$ Position Vector