

→ Centripetal Acceleration

The position vector at any time t is given by

$$\mathbf{r}(t) = r_0 \cos \omega t \mathbf{i} + r_0 \sin \omega t \mathbf{j} \quad \text{where } r_0 \text{ and } \omega \text{ are constants.}$$

Then the velocity vector $\mathbf{v}(t)$ is defined by

$$\mathbf{v}(t) = -r_0 \omega \sin \omega t \mathbf{i} + r_0 \omega \cos \omega t \mathbf{j}$$

and acceleration vector $\mathbf{a}(t)$ is defined by

$$\mathbf{a}(t) = -\omega^2 \mathbf{r}(t)$$

~~is correct~~

Let $v = \|v(t)\|$ and $a = \|a(t)\|$. Then

$$v = \|v(t)\| = \| -r_0 \omega \sin \omega t \mathbf{i} + r_0 \omega \cos \omega t \mathbf{j} \|$$

$$= \sqrt{(-r_0 \omega \sin \omega t)^2 + (r_0 \omega \cos \omega t)^2}$$

$$= \sqrt{r_0^2 \omega^2 \sin^2 \omega t + r_0^2 \omega^2 \cos^2 \omega t}$$

$$= \sqrt{r_0^2 \omega^2 [\sin^2 \omega t + \cos^2 \omega t]}$$

$$= \sqrt{r_0^2 \omega^2 \times 1}$$

$$[\because \sin^2 \omega t + \cos^2 \omega t = 1]$$

$$= \sqrt{r_0^2 \omega^2}$$

$$v = r_0 \omega$$

\implies

$$a = \|a(t)\| = \| -r_0 \omega^2 \cos \omega t \mathbf{i} - r_0 \omega^2 \sin \omega t \mathbf{j} \|$$

$$= \sqrt{(-r_0 \omega^2 \cos \omega t)^2 + (-r_0 \omega^2 \sin \omega t)^2}$$

$$= \sqrt{r_0^2 \omega^4 \cos^2 \omega t + r_0^2 \omega^4 \sin^2 \omega t}$$

$$= \sqrt{r_0^2 \omega^4 [\cos^2 \omega t + \sin^2 \omega t]}$$

$$= \sqrt{r_0^2 \omega^4 \times 1}$$

$$[\because \cos^2 \omega t + \sin^2 \omega t = 1]$$

$$= \sqrt{r_0^2 \omega^4}$$

$$a = r_0 \omega^2$$

$$a = \frac{r_0}{r_0} [r_0 \omega^2] = \frac{r_0 (r_0 \omega^2)}{r_0} = \frac{r_0^2 \omega^2}{r_0}$$

~~$a = \frac{r_0^2 \omega^2}{r_0}$~~

$$a = \frac{r_0^2 \omega^2}{r_0} = \frac{v^2}{r_0}$$

$$[\because v = r_0 \omega]$$

$$\therefore a = \frac{v^2}{r_0} \Rightarrow \text{centripetal Acceleration}$$

Motion of a Projectile

We know that $v'(t) = a(t)$. Hence integrating this equation with respect to t , we get $\int v'(t) dt = \int a(t) dt$

$$v(t) = \int a(t) dt$$

Let $v(0) = v_0$ be the initial condition. Then

$$v(t) = \int a(t) dt + v_0 \Rightarrow \text{Velocity Vector}$$

Since $v(t) = r'(t)$, we can obtain $r(t)$ by integrating both sides of the equation with respect to t . Thus we obtain

$$\int v(t) dt = \int r'(t) dt$$

$$\int r'(t) dt = \int v(t) dt$$

$$r(t) = \int v(t) dt$$

Let $r(0) = r_0$ be the initial condition. Then

$$r(t) = \int v(t) dt + v_0 t + r_0$$

$$r(t) = \int v(t) dt + v_0 t + r_0 \Rightarrow \text{Position Vector}$$

Find the velocity and position vector of an object with the given acceleration, initial velocity and position

$$a(t) = -32k, \quad v(0) = i + 2j, \quad r(0) = 128k \quad ?$$

$$\Rightarrow v_0 = i + 2j$$

$$r_0 = 128k$$

$$v(t) = \int a(t) dt = \int -32k dt = -32 \int dt = -32t k$$

~~$$v_0 = i + 2j, \quad v(t) = -32t k + i + 2j$$~~

~~$$\therefore v(t) = \int a(t) dt + v_0$$~~

$$v(t) = -32t k + i + 2j$$

$$v(t) = i + 2j - 32t k \Rightarrow \text{velocity vector}$$

$$r(t) = \int v(t) dt = \int -32t k dt = -32k \int t dt$$

$$r(t) = -32k \frac{t^2}{2}$$

$$r(t) = -16t^2 k$$

$$\therefore r_0 = 128k, \quad r(t) = \int v(t) dt + v_0 t + r_0$$

~~$$r(t) = \frac{1}{2} t^2 k$$~~

$$r(t) = -16t^2 k + (i + 2j)t + 128k$$

$$r(t) = t i + 2t j - 16t^2 k + 128k = t i + 2t j + (-16t^2 + 128)k$$

\Rightarrow Position Vector