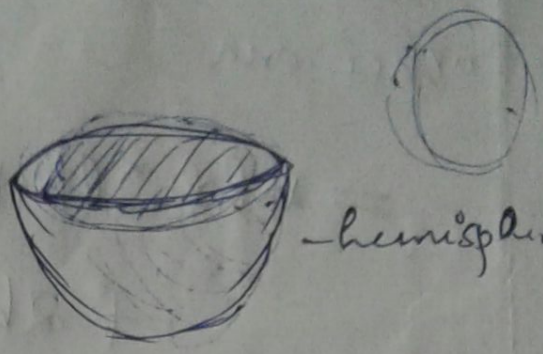
SURFACE AREAS AND VOLUMES

Sphere



Surface area of sphere $= 4\pi r^2$

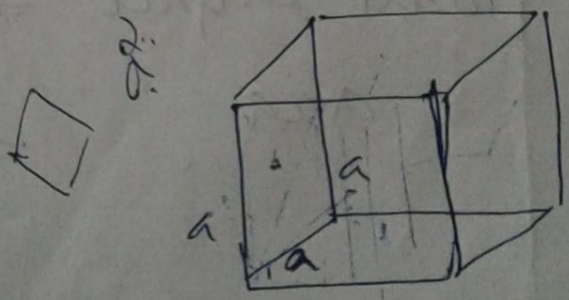
Curved Surface area of hemisphere $= 2\pi r^2$.

Total surface area of hemisphere $= 3\pi r^2$.

Volume of hemisphere $= \frac{2}{3}\pi r^3$.

Volume of sphere $= \frac{4}{3}\pi r^3$.

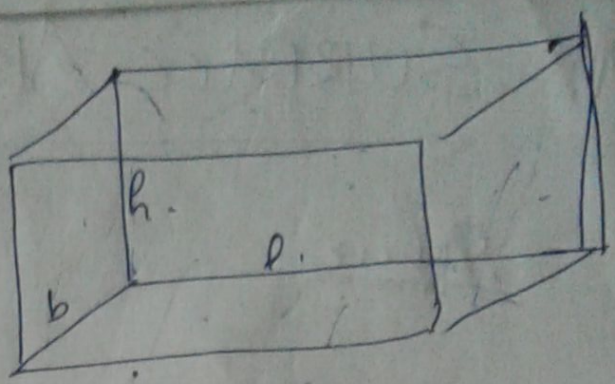
Cube



Surface area of sphere $= 6a^2$.

Volume $= a^3$.

Cuboid

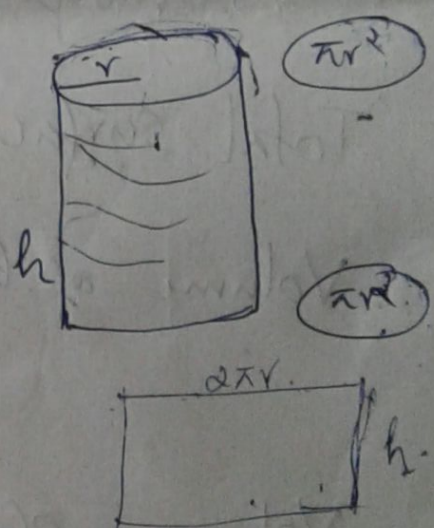


Surface area

$= 2lb + 2lh + 2bh$

Volume of cuboid $= l \times b \times h$.

Cylinder



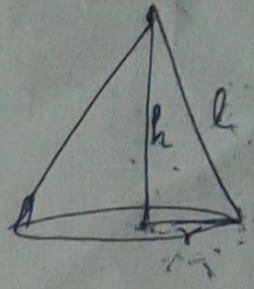
Curved surface area $= 2\pi rh$.

Total surface area $= 2\pi rh + 2\pi r^2$

$= 2\pi r(h+r)$

Volume $= \pi r^2 h$.

Cone



Curved Surface area $= \pi r l$.

h = height of cone
l = slant height
r = radius of circular surface

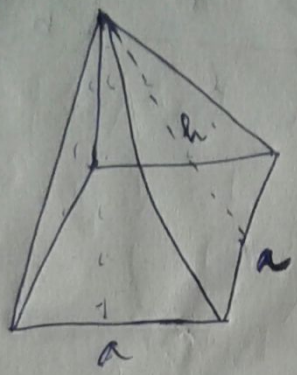
Total surface area $= \pi r l + \pi r^2$

Volume of cone $= \frac{1}{3} \pi r^2 h$.

$$l^2 = h^2 + r^2$$

$$l = \sqrt{h^2 + r^2}$$

Surface Area and Volume of Pyramid.

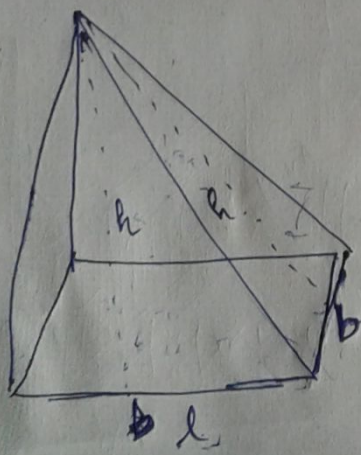


– A square pyramid.

Surface Area of Pyramid = Area of base + Lateral Surface Area.

$= a^2 + 4 \times \frac{1}{2} bh.$

$= a^2 + 2ah$



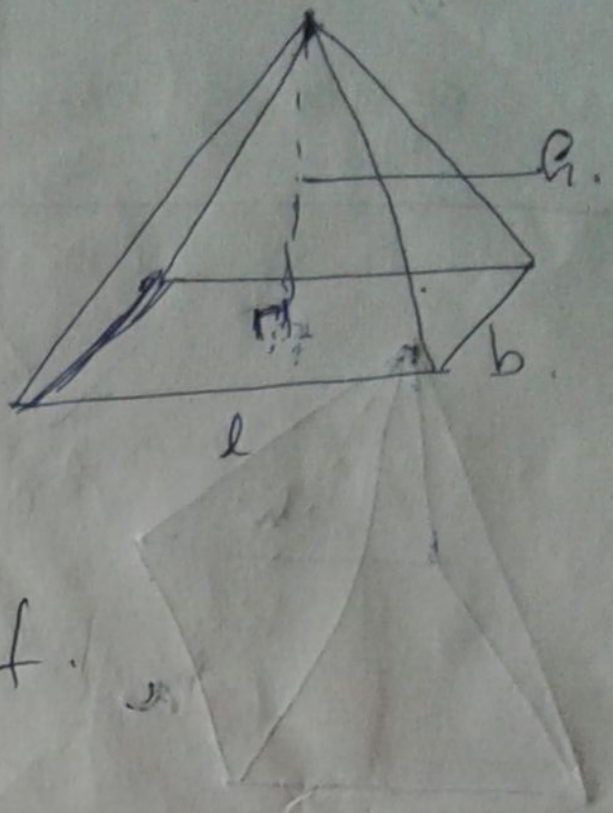
– A rectangular pyramid

S.A = Area of base + L.S.A

$= lb + 2 \times \frac{1}{2} bh + 2 \times \frac{1}{2} lh$

$= lb + bh + lh$

Volume of Pyramid



$$\text{Volume} = \frac{1}{3} A h$$

A = Area of base

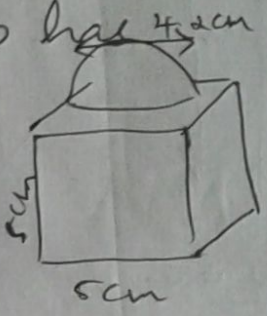
h - perpendicular height

$A = a^2$ - if square pyramid.

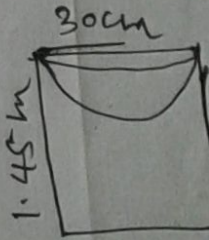
$A = l \times b$ - if rectangular pyramid.

Examples.

(1) A decorative block shown in Fig is made of a solids - a cube and a hemisphere. The base of the block is a cube with edge 5 cm and the hemisphere fixed on the top has a diameter of 4.2 cm.

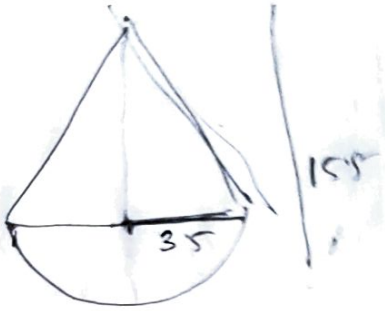


(2) Tony made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end. The height of cylinder is 1.45 m and its radius is 30 cm. Find the total surface area.



(3) 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

(4) A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the vessel.



T.S.A = CSA of hemisphere + CSA of cone.

$= 2\pi r^2 + \pi r l$.

Height of cone = $15.5 - 3.5 = \underline{12}$.

$l^2 = h^2 + r^2$

$l^2 = 12^2 + 3.5$

$= 144 + 12.25$

$= \underline{156.25}$

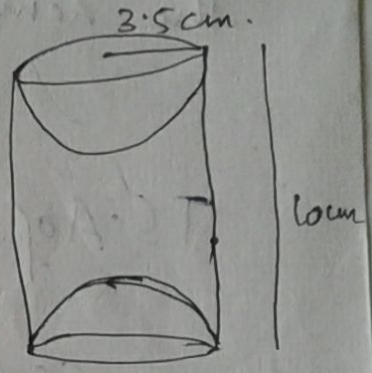
$l = \sqrt{156.25}$

$= \underline{12.5}$

$\therefore$ T.S.A $= 2 \times \frac{22}{7} \times 3.5^2 + \frac{22}{7} \times 3.5 \times 12.5$

$= 77 + 137.5 = 214.5 \text{ cm}^2$

(5) A wooden article was made by scooping out a hemisphere from each end of a solid cylinder. If height of the cylinder is 10 cm. and its base is of radius 3.5 cm, find the total surface area of the article.



T.S.A of article

= CSA of cylinder + 2 × C.S.A of hemisphere

$= 2\pi rh + 2 \times 2\pi r^2 = 2\pi rh + 4\pi r^2$

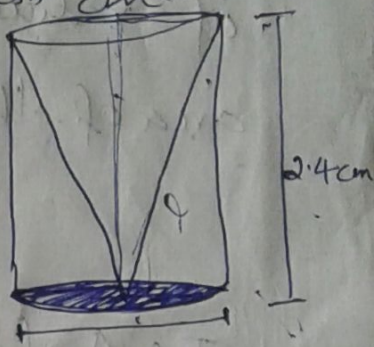
$= 2\pi r(h + 2r)$

$= 2 \times \frac{22}{7} \times 3.5 (10 + 7)$

$= 2 \times \frac{22}{\cancel{7}} \times \cancel{3.5}^{\,5} \times 17$

$= 374 \text{ cm}^2$

(6) From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.



T.S.A of solid =

CSA of cylinder +
CSA of cone +
Base area of cylinder.

$= 2\pi rh + \pi rl + \pi r^2$.

$= \pi r [2h + l + r]$

$=$

$l^2 = h^2 + r^2$

$l^2 = 2.4^2 + .7^2$

$= 6.25 \Rightarrow l = \sqrt{6.25} = 2.5$

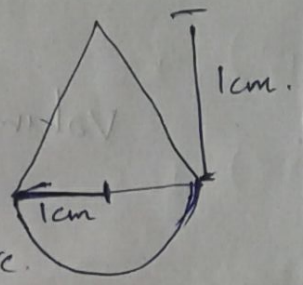
T.S.A of solid $= \frac{22}{7} \times 0.7 [2 \times 2.4 + 2.5 + 0$

$= 17.6 \ cm^2$

Volume

1)

(1) A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.



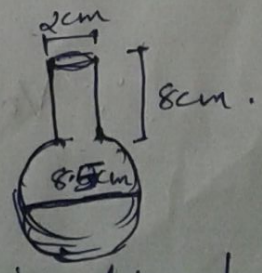
∴ Volume of solid = Volume of cone
+
Volume of hemisphere.

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi \times 1^2 \times 1 + \frac{2}{3} \times \pi \times 1^3$$

$$= \frac{1}{3}\pi + \frac{2}{3}\pi = \frac{3}{3}\pi = \pi \text{ //}$$

(2). A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter. the diameter of spherical part is 8.5 cm. By measuring the amount of water



water it holds, a child find its volume to be 345 cm³. Check whether is correct.

Volume of vessel = Volume of cylinder + volume of sphere

$= \pi r^2 h + \frac{4}{3}\pi r^3$

Volume of cylinder $= \pi r^2 h$, $r = 1$, $h = 8$

$= 3.14 \times 1^2 \times 8$

$= 25.12 \text{ cm}^3$.

Volume of sphere $= \frac{4}{3}\pi r^3$, $r = \frac{8.5}{2} = 4.25$

$= \frac{4}{3} \times 3.14 \times 4.25^3$

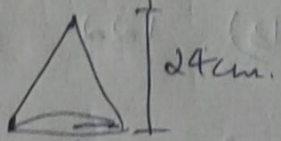
$= 321.39 \text{ cm}^3$

∴ Volum of vesel $= 321.39 + 25.12$

$= 346.5$

⇒ Almost correct.

(3) A cone of height 24 cm and radius of base 6 cm is made up of modelling clay. A child reshape it in the form of a sphere. Find the radius of the sphere.



∴ Volume of cone = Volume of sphere.

$$\frac{1}{3}\pi r^2 h = \frac{4}{3}\pi r^3$$

$$\frac{1}{3}\pi \times 6^2 \times 24 = \frac{4}{3}\pi r^3$$

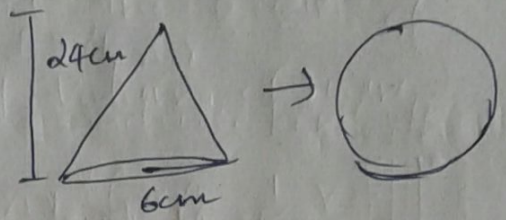
$$864 = 4r^3$$

$$r^3 = \frac{864}{4} = 216$$

$$r = \sqrt[3]{216} = 6$$

∴ Radius of sphere = 6 cm

(4) A cone of height 24 cm and radius of base 6 cm is made-up of modelling clay. A child shape it in the form of a sphere. Find the radius of the sphere.



∴

Since cone is remodelled into sphere

Volume of cone = Volume of sphere.

Volume of cone $= \frac{1}{3}\pi r^2 h$

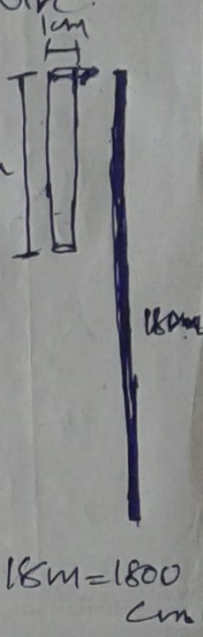
$= \frac{1}{3} \times \frac{22}{7} \times 6^2 \times 24$

$= 903.14$

Volume of sphere $= \frac{4}{3}\pi r^3$

(4) A copper rod of diameter 1 cm and length 8 cm is drawn into a wire of length 18 km of uniform thickness. Find the thickness of wire.

∴ Volume of rod = Volume of wire



$$\pi r_1^2 h_1 = \pi r_2^2 h_2$$

$$\pi \times \left(\frac{1}{2}\right)^2 \times 8 = \pi r_2^2 \times 1800$$

$$\frac{1}{4} \times 8 = r_2^2 \times 1800$$

$$2 = r_2^2 \times 1800$$

$$r_2^2 = \frac{2}{1800}$$

$$r_2 = \sqrt{\frac{2}{1800}}$$

$$= \sqrt{\frac{1}{900}} = \frac{1}{30}$$

The thickness of wire = diameter of wire

$$= \frac{1}{30} \times 2$$

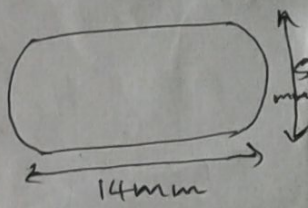
$$= \frac{2}{30} = \frac{1}{15} \text{ cm}.$$

(5) A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

(6) A 20 m deep well with diameter 7 m is dug and earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

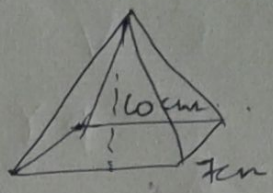
1) Find the area of a rectangular park whose perimeter is 480 m, 80m.

2) A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm and diameter of the capsule 5mm. Find surface area



3) A cylindrical bucket 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of conical heap is 24cm., find the radius of the conical heap.

(1) Calculate the surface area of pyramid with a square base of side 7 cm and height of triangular side is 10 cm.



T.S.A = Area of base
+
4 × Area of triangular side

$= a^2 \times 4 \times \frac{1}{2} \times b \times h$

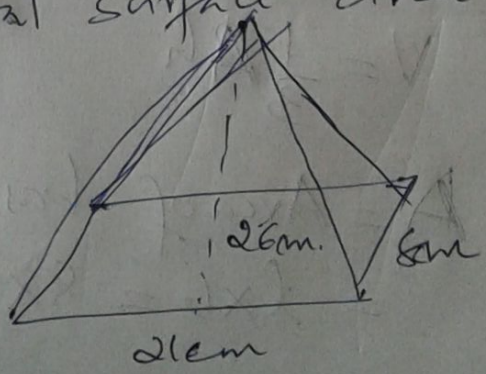
$= a^2 \times 2bh$

$= 7^2 \times 2 \times 7 \times 10$

$= 49 + 140$

$= 189\ cm^2$

(2) Calculate the total surface area of the given pyramid.



T.S.A = Area of base
+
Area of 4 triangular sides.

Area of base $= l \times b$

$= 21 \times 8$

$= \underline{168 \text{ cm}}$.

Area of triangular sides $= 2 \times \frac{1}{2} bh + 2 \times \frac{1}{2} lh$

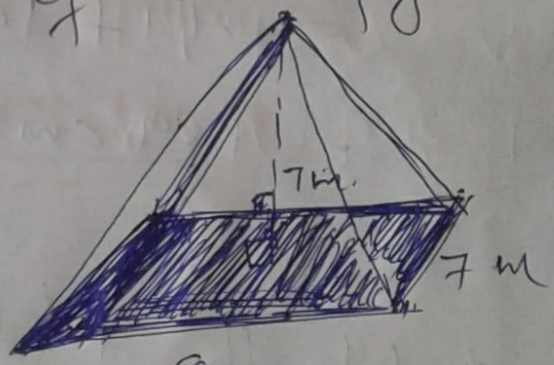
$= bh + lh$

$= 21 \times 26 + 8 \times 26$

$= 754 \text{ cm}^2$

T.S.A $= 754 + 168 = \underline{922 \text{ cm}^2}$

(3) Find volume of the pyramid.



$V = \frac{1}{3} Ah$

A = Area of base

$= l \times b = 8 \times 7 = \underline{56} \text{ m}^2$.

Volume $= \frac{1}{3} Ah = \frac{1}{3} \times 56 \times 7$

$= \underline{130.667 \text{ m}^3}$